

## The steady-state format of global climate

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### SUMMARY

It can be proved for certain small-scale convective heat transfer processes that the preferred steady-state mode is one of maximum entropy production. The constraint is more or less equivalent to one of maximum kinetic energy dissipation or of maximum convective heat transport. Evidence is accumulating that the same constraint may apply on the much larger scale of the earth-atmosphere system. The concept is accepted as the basis of a purely thermodynamic model of the mean annual global climate. The model allows *a priori* calculation of the broad-scale geographic distributions of cloud, surface temperature, horizontal energy fluxes in the ocean and in the atmosphere, net radiant energy inputs, etc. The agreement with observation strongly supports the basic concept. It suggests also that, to the extent allowed by the degrees of freedom in the dynamics, the partition of atmospheric and oceanic energy flow is determined by a requirement to equalize the local dissipations in the two media.

### 1. INTRODUCTION

Various extremum principles have been suggested which might allow prediction of the broad format of global climate without the necessity for complete description of the internal workings of the system. Lorenz (1960) surmised that the atmosphere is constrained to operate in a mode close to a maximum in the efficiency defined as the ratio of kinetic energy production to solar energy input (see also Schulman 1977). Dutton and Johnson (1967) proposed that atmospheric motions obey a principle of least action. They defined 'action' as the integral of the difference between the local kinetic and total potential energy densities. Dutton (1973) introduced the concept of 'entropic energy' – a measure of the departure of the atmosphere from the motionless, hydrostatic and isothermal condition towards which an atmosphere would naturally tend when isolated from energy input. He suggested that real atmospheric behaviour minimizes entropic energy. Paltridge (1975) developed a purely thermodynamic zonal average model of the global system, and on the basis of its behaviour suggested a constraint of minimum entropy exchange.

If any or all of the suggested constraints have a physical basis, that basis must surely have roots in the thermodynamics of the steady state. Unfortunately, the discipline has yet to make significant advances in the study of nonlinear processes where the transfer coefficients are themselves functions of the fluxes (see, however, Glansdorff and Prigogine 1971). It has been highly successful in the analysis of linear systems. For these Prigogine (1968) has shown that a principle of minimum entropy production is certainly applicable, and that it can be used as a predictor of steady-state format. There are restrictions on its use (de Groot and Mazur 1963) but it has been argued by Gyarmati (1970) that a form of the principle may provide the basis of one of the most powerful laws of physics. He suggests that many of the extremum principles of physics can be restated in terms of minimization of an appropriately defined entropy production rate.

It is a matter of speculation whether such a broad generalization will emerge to cover nonlinear processes, or whether in any event it would be mathematically provable. The most that can be done in the present context of the earth-atmosphere system is to seek for possible extremum constraints and attempt on the one hand to justify them individually (by



Suppose that the system was inherently linear in that

$$X = k dT/dx \quad (5)$$

where  $k$  is some constant transfer coefficient. During the approach to steady state the Prigogine law would require that  $\dot{S}_T$  of Eq. (3), and  $D$  of Eq. (4), approach their minimum values compatible with external conditions. In any format away from steady state, the time rates of change,  $dD/dt$  or  $d\dot{S}_T/dt$ , would be negative.

Consider the more realistic nonlinear situation where the transfer coefficient,  $k$ , is unconstrained to the extent that it can adopt one of a number of values corresponding to the allowable modes of the system dynamics. In fact, for purposes of illustration suppose  $k$  can adopt any of a continuous spectrum of values from zero to infinity. There is an infinite number of possible steady-state formats for such a system. The question arises whether there may be preferential selection of a particular one of that number. On no more than the grounds of uniqueness, one might legitimately wonder whether the steady-state mode of maximum dissipation or entropy production has some significance. Other than for equilibrium isothermal situations, minimum (i.e. zero) dissipation would correspond to the physically untenable extreme states where either  $X = 0$  (zero  $k$ ) or  $dT/dx = 0$  (infinite  $k$ ).

It has been proved for a few cases of turbulent energy flow where multiple steady states are possible, that a principle of maximum dissipation applies to mode selection. Busse (1967) has proved this to be the case for certain convection processes under the condition (among others) of constant Rayleigh number (see also Palm 1972). This in turn is equivalent to the selection of a mode which maximizes efficiency of convective heat transport (Malkus and Veronis 1958). Unfortunately, the proofs of such phenomena cannot be applied under certain conditions. The most notable in the present context are those conditions pertaining to rotating systems.

The global-scale constraints of Lorenz and of Paltridge are equivalent in that they both suggest the earth-atmosphere is in a format which maximizes the rate of entropy production. At steady state the kinetic energy production is equal to the kinetic energy dissipation. Bearing in mind that kinetic energy dissipation is the only significant irreversible process in the atmosphere (Lorenz 1967), the constraint of Lorenz is indeed one of maximum total dissipation or entropy production. Paltridge suggested a minimum of the inwardly directed entropy exchange,  $\dot{S}_e$ , which from Eq. (1), and since it is negative, is also equivalent to maximum entropy production.

Thus there is encouragement to pursue the idea that the earth-atmosphere system obeys a basic extremum principle of maximum dissipation or entropy production. The realistic situation would involve preferential selection of one of a number of possible steady-state formats. It is assumed here that there are sufficient degrees of freedom in the dynamics and thermodynamics of the system to allow virtually any format which satisfies energy balance and boundary conditions.

Climate variations occur on all scales, but the earth-atmosphere is at least close to a form of steady state in that the broad annual mean parameters of climate are definable and fairly constant with time. On the other hand the seasonal variation of solar position and the heat storage of the oceans ensure that the system is never in the 'pure' stationary condition, SS, which would pertain if the sun were continuously at its mean position over the equator. The thermodynamic concepts above concern pure steady states. They are not likely to be applicable to a format of the real system at some particular time of year. Therefore the present work compares a thermodynamic model of a theoretical world in the condition SS with the observed (and broad-scale) annual mean format of actual climate. The assumption

is made that the observed annual means are close to those which would pertain under the condition SS.

The model envisages the earth-atmosphere as a classic 'closed' steady-state system which can exchange energy but not matter with its environment. The system boundary is at the top of the atmosphere, so that by reference to Eq. (1)

$$\dot{S}_I = -\dot{S}_e = -\int (dQ/T_a) dA \quad (6)$$

Here  $dQ$  is the net rate of radiant energy input to the area element  $dA$  of boundary surface; the integral (in practice, the sum) is taken over the entire boundary; and  $T_a$  by classic definition should be the temperature of the medium at the boundary. Because radiant energy can be absorbed or emitted at the ground and at various levels of the atmosphere, there can be some argument with the classic definition. Suffice it to say that the measure of  $T_a$  chosen here (which is at least based on the classic idea) gives good results. Other concepts of boundary temperature (involving, say, the temperature at which the radiation streams are absorbed) do not.

In the model,  $T_a$  and the solar and infrared fluxes which make up  $dQ$  are known functions of cloud cover,  $\theta$ , and surface temperature,  $T$ , of each region. In turn  $\theta$  and  $T$  are functions of the distribution of horizontal energy flows in the atmosphere and ocean – a distribution which is free to adopt any format provided energy balance is maintained everywhere on the globe. There are no heat storage terms in the equations and no dynamic restrictions on possible energy flows. The model is free to adopt any of an infinite set of steady-state formats. It is designed to select that distribution of horizontal energy flows which satisfies a constraint of maximizing the  $\dot{S}_I$  of Eq. (6).

Before proceeding, it may be said that the constraints of Dutton and of Dutton and Johnson mentioned earlier concern not so much the rates of production of a quantity but rather the extremum value of the quantity itself. In principle their constraints are compatible with the previous discussion. By analogy with linear theory it might be argued (not very successfully and on philosophical grounds only) that the approach is perhaps not as fundamental.

### 3. THE ZONAL AVERAGE MODEL

The overall concept is similar to that of I. It concerns a zonal average model consisting of 10 adjacent 'boxes' covering the entire globe from north to south pole. Each box has equal surface area, and mean and limiting latitudes are recorded in Table 1. There are eighteen cross-box meridional flows of energy – nine atmospheric and nine oceanic flows referred to as  $X_{ai}$  and  $X_{oi}$  respectively ( $i = 1$  to 9). These flows are the basic unknowns of the problem. The object is to express the total entropy production,  $\dot{S}_I$ , of the planet as a function of the  $X_{ai}$  and  $X_{oi}$  and to find that unique set which leads to a maximum in  $\dot{S}_I$ . To this end the entropy exchange with space of each box, and hence its contribution to the total entropy exchange of the planet with space, is expressed as a function of the local convergences,  $\Delta X_{ai}$  and  $\Delta X_{oi}$ , of the meridional energy flows. The position of the minimum in total entropy exchange,  $\dot{S}_e$ , and therefore by Eq. (1) the position of the maximum in  $\dot{S}_I$ , is found and specified in terms of a set of the local convergences. Here and in the following the boxes are numbered from the north to the south pole; this direction is regarded as positive, and flow  $X_i$  corresponds to the flow through the southern boundary of box  $i$ . Since  $\Delta X_{a,i} = X_{a,i-1} - X_{a,i}$  and  $\Delta X_{o,i} = X_{o,i-1} - X_{o,i}$ , the actual flows can be found from the convergences.

TABLE 1. CONSTANTS REQUIRED FOR THE OPERATION OF THE MEAN ANNUAL ZONAL AVERAGE MODEL (SEE TEXT FOR SYMBOL DEFINITIONS). NOTE THAT IN I,  $f$  WAS MISQUOTED AS 0.3

	N.P.		Eq.		S.P.	
Lat. (mean) (°)	64.0		17.4	5.7	17.4	64.0
Lat. (lim.) (°)	90.0		23.5	11.5	23.5	90.0
$\alpha$	0.15	0.12	0.09	0.08	0.08	0.14
$\theta_0$	0.13	0.10	0.08	0.08	0.08	0.13
$d_0$	0.41	0.35	0.33	0.32	0.33	0.41
$f_0$		0.30	0.60	0.75	0.80	0.95

Other constants:  $R_0 = 136.0 \text{ mW cm}^{-2}$ ;  $\bar{g} = 0.1$ ;  $\bar{d} = 0.35$ ;  $m = 0.14$ ;  $a = 0.04$ ;  $\bar{m} = 0.2 \text{ m}$ ;  $f = 0.8$ ;  $G = 0.4$ ;  $\varepsilon = 0.3$ ;  $\varepsilon' = 0.75$

(a) *The individual box*

Each box consists of an atmosphere and an ocean envisaged as a system for which there are two independent energy balance equations and three unknowns, once the convergences of atmospheric and oceanic meridional energy flows are specified. The unknowns are cloud cover,  $\theta$ , surface temperature,  $T$ , and the ocean-to-atmosphere flux,  $LE + H$ , of latent and sensible heat ( $LE$  and  $H$  respectively).

Total energy balance of a latitude zone requires that

$$\{[R_0 - gR_0(1 - \theta) - dR_0\theta]/\xi\} - \{\varepsilon\sigma T^4(1 - \theta) + G\sigma T^4(1 - \theta) + f\varepsilon'\sigma T^4\theta\} + \Delta X_a + \Delta X_o = 0 \quad (7)$$

The first term in curly brackets is the net shortwave flux,  $F_{\downarrow s}$ , into the top of the atmosphere of the zone. Within this term  $R_0$  is the solar constant,  $g$  is the planetary albedo of the cloud-free portions of the globe,  $d$  is the planetary albedo of those portions which are cloud covered, and  $\xi$  is the ratio of the actual surface area to the projected area as seen by the sun. The second term in curly brackets is the longwave flux,  $F_{\uparrow L}$ , returned to space, partitioned into three separate contributions all of which are parameterized as a function of the black-body radiation  $\sigma T^4$  corresponding to surface temperature  $T$  (Stefan-Boltzmann law);  $\varepsilon$  is a fraction related to the width of the window region of the black-body spectrum ( $\sim 7.5$ – $12.5 \mu\text{m}$ ) where direct exchange of radiation between surface and space is possible;  $G$  is a fraction specifying the longwave loss of the cloud-free atmosphere to space as a function of surface black-body emission;  $f$  is the fraction by which black-body radiation from clouds is reduced below surface black-body emission by reason of their lower temperature; and  $\varepsilon'$  is a constant related to the width of the window region of the infrared spectrum above the bulk of the atmospheric water vapour.  $\Delta X_a$  and  $\Delta X_o$  are respectively the convergences of atmospheric and oceanic meridional energy flow.

Oceanic energy balance of a latitude zone requires that

$$\{[R_0 - gR_0(1 - \theta) - (d + a)R_0\theta - R_0m]/\xi\} - \{\varepsilon\sigma T^4 - f\varepsilon\sigma T^4\theta\} - (LE + H) + \Delta X_o = 0 \quad (8)$$

Here the first term in curly brackets is the net shortwave flux into the surface. It takes into account the extra shortwave absorption,  $a$ , attributable to liquid water in clouds and the absorption,  $m$ , by water vapour in the atmosphere. The second term in curly brackets is the net upward longwave flux from the surface. It consists of the total upward flux through the window region and the downward flux returned through the window from clouds. Net longwave exchange at the surface outside the window region is assumed to be zero (Swinbank 1963).

To this point the equations are identical to the description in I except for the mode of the explicit separation of cloudy region albedo and the cloudy region extra solar absorption. The parameterization of the longwave fluxes in terms of  $\sigma T^4$  is arguable, but is equivalent to the assumptions necessary in any simple 'box model' of the atmosphere.

A shortcoming of the physics of the two equations is that they contain no feedback between planetary albedo ( $g$  and  $d$ ) and atmospheric absorption ( $m$ ). In the real situation an increased absorption must reduce the solar energy available for reflection and hence must reduce the planetary albedo. The equations were recast in terms of surface albedo  $\alpha$ . In Eq. (7),  $g$  and  $d$  were replaced with  $g_p$  and  $d_p$  respectively, where

$$\left. \begin{aligned} g_p &= g_0 + \alpha - \alpha(g_0 + \bar{g} + m + \bar{m}) \\ d_p &= d_0 + \alpha - \alpha(d_0 + \bar{d} + m + \bar{m}) \end{aligned} \right\} \quad (9)$$

and

In Eq. (8),  $g$  and  $d$  were replaced with  $g_G$  and  $d_G$  respectively, where

$$\left. \begin{aligned} g_G &= g_0 + \alpha \\ d_G &= d_0 + \alpha + a \end{aligned} \right\} \quad (9a)$$

and

These expressions are first-order representations of an atmosphere where specific account is taken of the following: planetary albedo,  $g_0$ , of the clear-sky Rayleigh atmosphere to the solar beam; planetary albedo,  $d_0$ , of the cloudy atmosphere to the direct solar beam; albedo,  $\bar{g}$ , of the Rayleigh atmosphere to diffuse upward flux reflected from the ground; albedo,  $\bar{d}$ , of the cloudy atmosphere to diffuse upward flux reflected from the ground; and atmospheric absorption,  $\bar{m}$ , of diffuse flux reflected from the ground.

A somewhat artificial device was employed in I in order to obtain a third energy balance equation required for solution of the three unknowns. The device involved an assumption about the partition of the deposition of  $LE$  and  $H$  into the upper and lower regions of the atmosphere respectively. It was one of the most unhappy features of the paper, and the following arose from efforts to remove it.

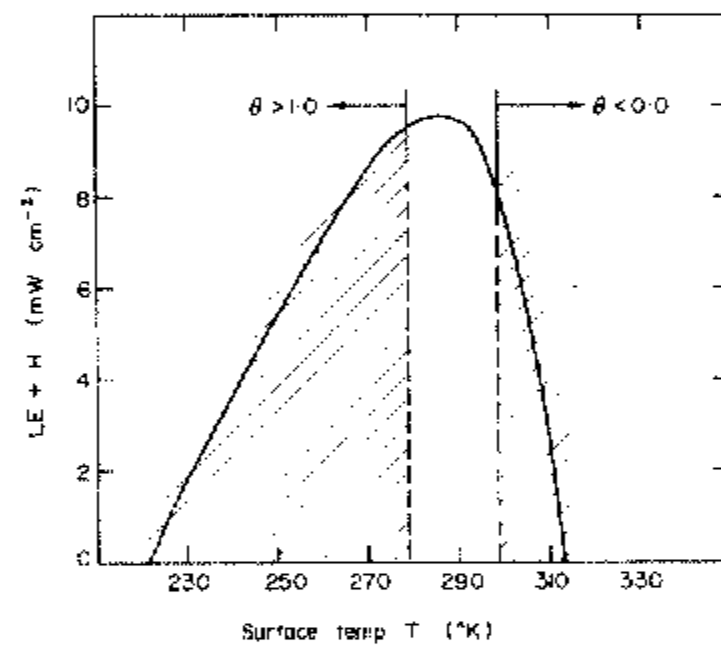


Figure 2. The ocean-to-atmosphere vertical heat flux,  $LE+H$ , as a function of surface temperature for the global mean conditions recorded in the caption to Table 2. These are the values allowed by energy balance of planet and surface (Eqs. (7) and (8)). Note that the regions outside the range  $0 \leq \theta \leq 1$  are physically unreal.

$LE+H$  is a function of the other unknowns  $\theta$  and  $T$  from Eq. (8). It can be shown from Eqs. (7) and (8) that, for fixed  $\Delta X_a$  and  $\Delta X_o$ , the function has a single maximum in the 'surface' of the physically real ranges of  $\theta$  and  $T$ . (See Fig. 2 where  $LE+H$  is plotted as a function of  $T$  by combining the equations to eliminate cloud cover,  $\theta$ .) Further, the maximum occurs at more or less the observed values of cloud cover and surface temperature for each of the latitude zones when the observed meridional energy convergences,  $\Delta X_a$  and  $\Delta X_o$ , are used in the solution. It emerges later in the results of Fig. 3 that this is so.

On this evidence it would appear that the upward turbulent flux of latent and sensible heat may conform to the suggestion of Malkus and Veronis that mode selection is determined by a tendency to maximum efficiency of convective heat transport. In view of the involvement of latent heat, the mechanical means by which the result might be achieved is obscure except that, other things being equal, an  $LE+H$  less than its maximum value would tend to increase the temperature difference between whatever is the appropriate mean atmospheric temperature and that of the ground. This in turn would decrease the stability and increase the relevant transfer coefficients. Suffice it to say that the feature is compatible with the broad principle of maximum dissipation which is the general thesis of this paper, and is therefore used in the following as a means of obtaining unique values of the three variables  $\theta$ ,  $T$  and  $LE+H$  from Eqs. (7) and (8). In principle the constraint supplies a third

equation. In practice the values of  $\theta$  and  $T$  which satisfy Eqs. (7) and (8) and which yield a maximum in  $LE+H$  are computed by numerical iteration.

It is possible at this point to examine the sensitivity of a zone to changes in any of the constants of the zone provided the meridional energy flux convergences are specified. It is possible also to examine the sensitivity to meridional convergence. The results of such calculations are given in Table 2 for global average conditions (or for a zone whose parameters are equal to the global averages) and will be discussed later. At this time note that  $\theta$  and  $T$ , and hence the solar input  $F\downarrow_s$  and longwave output  $F\uparrow_L$  of a zone, are dependent on the total convergence of meridional energy flux but are independent of the ratio of  $\Delta X_a$  to  $\Delta X_o$ . This is numerically apparent when the results of Table 2 are interpreted in terms of a single zone. It becomes mathematically apparent from a manipulation of Eqs. (7) and (8) in order to obtain specific expressions for  $\partial(LE+H)/\partial T$  and  $\partial(LE+H)/\partial\theta$ . Both these partial derivatives, and hence the point at which they are simultaneously zero as required by the constraint, depend only on the sum of  $\Delta X_a$  and  $\Delta X_o$ . Variations in the ratio  $\Delta X_a/\Delta X_o$  are reflected only in changes of  $LE+H$ . The point is important in the following.

(b) *The overall model*

Assembly of the 10 boxes into a complete planetary model enables the total entropy production to be calculated by computer for any given set of the  $X_{ai}$  and  $X_{oi}$ . From Eq. (1)  $\dot{S}_I = -\dot{S}_e$ , and  $\dot{S}_e$ , the total rate of exchange of entropy of the planet with space, can be calculated as the sum of the contributions from each box. That is,

$$\dot{S}_I = -\dot{S}_e = -\sum_{i=1}^{10} (F\downarrow_s - F\uparrow_L)_i / T_{ai} \quad (10)$$

where the  $(F\downarrow_s - F\uparrow_L)_i$  are the net flows of radiant energy into the upper boundaries of the boxes and the  $T_{ai}$  are the temperatures of those boundaries.  $F\downarrow_s$  and  $F\uparrow_L$  are known functions of cloud cover and surface temperature (the first and second terms, respectively, of Eq. (7)) and these in turn can be calculated for each box according to the previous discussion once all the  $X_{ai}$  and  $X_{oi}$  are specified. Definition of boundary temperature is a problem since the atmosphere is not uniform in the vertical. Any simple model is faced with assignation of suitable averages to the vertically varying quantities. Here it is assumed that a single average temperature can be assigned to the atmosphere which is usable in all contexts of dissipation associated with the total vertical and horizontal fluxes of energy. Among these contexts is that of upper boundary temperature. Further, the  $T_{ai}$  are defined in terms of longwave emission by naïve application of the Stefan-Boltzmann law:

$$T_a = \{[G\sigma T^4(1-\theta) + f\epsilon'\sigma T^4\theta]/\sigma\}^{\frac{1}{4}} \quad (11)$$

where it can be seen by reference to Eq. (7) that only the atmospheric contribution to the total  $F\uparrow_L$  is included. (In I the equivalent temperature definition contained also the flux emanating from the ground so that  $T_a$  was simply  $(F\uparrow_L/\sigma)^{\frac{1}{4}}$ .) The Stefan-Boltzmann fourth-power definition of  $T_a$  is arguable but convenient. The important feature of Eq. (11) is that  $T_a$  has the qualitatively correct property of increasing with increasing radiant energy output from the atmosphere and hence with balancing energy input. It is at least a proportional measure of the temperature of that medium. Further, again in contrast to I, the definition gives to the model the property that variations of  $T_a$  and  $T$  can be of opposite sign – as can stratospheric temperature (or the effective radiative temperature of the absorbing gases in the troposphere) and surface temperature in the real world.



TABLE 2. PART 1 - SENSITIVITIES OF  $\theta$ ,  $T$ ,  $LE+H$ , and  $T_a$  TO CHANGES IN THE PARAMETERS LISTED, WHEN THE HORIZONTAL ENERGY CONVERGENCES,  $\Delta X_a$  AND  $\Delta X_o$ , ARE HELD CONSTANT. PART 2 - THE SENSITIVITIES TO  $\Delta X_a$  AND  $\Delta X_o$ . UNITS OF  $T$  AND  $T_a$  ARE DEGREES K; UNITS OF  $LE+H$  ARE  $mW\,cm^{-2}$ . NOTE: THE SENSITIVITIES ARE COMPUTED FOR A GLOBAL MEAN WORLD (OR FOR A ZONE WHOSE PARAMETERS ARE THOSE OF THE GLOBAL MEAN). THAT IS FOR:  $R_o = 136.0$ ,  $\alpha = 0.12$ ,  $m = 0.14$ ,  $g = 0.10$ ,  $\bar{g} = 0.10$ ,  $d_o = 0.35$ ,  $\bar{d} = 0.35$ ,  $a = 0.04$ ,  $\varepsilon = 0.30$ ,  $\varepsilon' = 0.75$ ,  $f = 0.80$ ,  $G = 0.40$ ,  $\xi = 4.0$ ,  $\Delta X_a = 0.0$ ,  $\Delta X_o = 0.0$ . WITH THESE PARAMETERS, PREDICTED VALUES ARE  $\theta = 0.505$ ,  $T = 286.584$ ,  $LE+H = 9.942$ ,  $T_a = 228.553$ ,  $F\downarrow_s = 24.840$ ,  $F\uparrow_s = 24.840$ ,  $F\downarrow_L$  AND  $F\uparrow_L$  ARE IN  $mW\,cm^{-2}$

Parameter ( $p$ )	$\delta\theta/\delta p$	$\delta T/\delta p$	$\delta(LE+H)/\delta p$	$\delta T_a/\delta p$	$\delta p$	$\delta\theta/\delta\ln p$	$\delta T/\delta\ln p$	$\delta T_a/\delta\ln p$
Part 1	$\alpha$	-0.93	0.60	-2.86	-20.35	0.10	0.72	-24.42
	$g_o$	1.41	-13.47	-1.13	6.96	0.10	-13.47	6.96
	$\bar{g}$	0.10	-0.19	-0.05	0.71	0.10	-0.19	0.71
	$d_o$	-1.75	9.36	-1.04	-20.07	0.10	32.76	-70.95
	$\bar{d}$	-0.03	0.79	-0.05	-0.45	0.10	2.76	-1.57
	$m$	0.10	0.71	-3.53	-2.71	0.10	9.94	-3.79
	$a$	-2.00	15.67	-2.12	-15.70	0.10	6.27	-6.28
	$\varepsilon$	1.05	-11.99	-1.66	4.24	0.10	-35.97	12.72
	$\varepsilon'$	0.09	-5.47	0.42	2.03	0.10	-41.02	15.22
	$f$	0.43	-9.19	1.09	4.26	0.10	-73.52	34.08
	$G$	-0.57	1.84	0.27	-3.92	0.10	7.36	-15.68
	$R_o$	0.0	7.10	1.02	5.67	+10%	71.00	56.70
	$\xi$	0.0	-15.50	-1.99	-12.40	1.0	-62.00	-49.60
Part 2	$\Delta X_a$	0.20	1.46	-0.24	4.65	1.0		
	$\Delta X_o$	0.20	1.46	+0.76	4.65	1.0		

In any event the  $T_{ai}$  are calculable functions of the  $X_{ai}$  and  $X_{oi}$  via their dependence on cloud cover and surface temperature in Eq. (11). As is the case for the solar and longwave output of a zone, the atmospheric temperature is dependent only on the total meridional energy flux convergence and not on the ratio  $\Delta X_{ai}/\Delta X_{oi}$  (see Table 2, for instance). Thus the ratio of the actual flows,  $X_{ai}/X_{oi} (= R_i)$ , is a degree of freedom which has no effect on planetary entropy exchange. The position of the point of maximum entropy production can be specified in terms only of the nine cross-box flows of total meridional energy flux,  $(X_a + X_o)_{i=1,9}$ . This is done here using, as in I, the simplex method of Nelder and Mead (1965) to find the position of maximum in  $\dot{S}_I$  (minimum in  $\dot{S}_o$ ) as defined by Eq. (10).

It might be argued that the whole problem could have been set up as a purely one-dimensional affair without specific separation of atmosphere and ocean. Quite apart from the inherent desire to predict surface temperature,  $T$ , rather than some vague measure of atmospheric or planetary temperature, the outgoing longwave flux is directly controlled by  $T$  via emission through the atmospheric window.  $T$  must be included specifically.

(c) *The ratio,  $R$ , of atmospheric to oceanic meridional flux*

The most important feature of the freedom associated with the  $R_i$  is that it allows a simple, basically thermodynamic, argument for the partition of the meridional energy flows.

The temperature structure (i.e. the horizontal variation of  $T_i$  and  $T_{oi}$ ) is set by the overall constraint involving the sums of  $X_{ai}$  and  $X_{oi}$ . Imagine a situation at a given latitude where  $X_a$  is zero – the extreme situation earlier referred to where the atmospheric transfer coefficient,  $k_a$ , had selected zero from the infinite (?) number of possible values. The total meridional flux is carried by the ocean and all the dissipated energy –  $X_o dT/T$  is generated there. This energy will be deposited in the atmosphere. It will be deposited in the form of energy convertible to atmospheric motion, which on physical grounds will ensure a non-zero  $k_a$  and a non-zero  $X_a$ . The energy now dissipated in the atmosphere ( $-X_a dT_a/T_a$ ) can radiate directly to space. The increase in  $X_a$  will continue at the expense of  $X_o$  until the radiation of dissipated energy to space exactly balances the deposition from the ocean, at which point the rates of dissipation in the two media will be identical. This then is the mechanism proposed as governing the partition of atmospheric and oceanic energy flow: that to the extent allowed by the dynamics of the system (there are no such limitations in the present model) the system adopts a format which equalizes the local dissipations in the two media. If  $\hat{\epsilon}$  is a small increment in horizontal energy flux, one can envisage a perturbation from the position of equal dissipation such that  $X_a$  increases to  $X_a + \hat{\epsilon}$  and therefore  $X_o$  decreases to  $X_o - \hat{\epsilon}$ . The resultant net transfer of dissipated energy across the atmosphere-ocean boundary cannot involve the deposition of heat since this would require an alteration of ocean temperature. The energy must be deposited either directly or ultimately in the form of kinetic energy, leading to an increase of  $X_o$  and a reduction of the perturbation to zero. Advection of the energy deposited in the ocean away from the local area amounts to the same thing – an increase of  $X_o$ , a consequent reduction of  $X_a$  and a return to zero of the net flux of *dissipated* energy across the boundary.

The above involves a physical assumption which cannot here be justified completely. It amounts to a statement that, if it is dynamically and thermodynamically possible, dissipated energy will be deposited across a fluid boundary in the form of kinetic energy. (The present model is peculiar in that it has both the dynamic and thermodynamic freedom for such to be the case.) The concept is not entirely new. Ramsey (1935), for instance, had recourse to a vaguely similar device in his discussion of pressure forces at a fluid boundary.

The definition of 'local' will of course depend on the scale of the dynamic features and the number of degrees of freedom allowed to them. Suffice it to say that for this model

$$f_0(X_a dT_a/T_a) = X_o dT/T \quad (12)$$

where  $f_0$  is the fraction of the latitude circle occupied by ocean (see Table 1). Since  $X_a + X_o$  and the temperature structure are specified by the overall constraint of maximum entropy production, Eq. (12) can be used to partition the total horizontal flow into the separate components  $X_a$  and  $X_o$ .

#### (d) Results

Table 1 contains the zonal average constants used as input data for the problem. Of the quantities dependent on solar angle, only surface albedo,  $\alpha$ , is non-symmetrical about the equator. The values of  $\alpha$  are from Sellers (1965). Values of the other parameters derive from a number of sources (see I) and, where there may be some argument as to an appropriate value, a policy of using round numbers has been pursued. With the same philosophy, the latitude-dependent variables  $g_0$  and  $d_0$  have been tuned so that their mean values are equal to  $\bar{g}$  and  $\bar{d}$ . A case could be made for including a latitude dependence of the constants controlling the infrared streams, but additional and doubtful complexity here would do no more than obscure the central issue. Note that  $G$  has been increased to 0.4 (cf. the 0.3 of I) to ensure that  $\partial F_{\uparrow L}/\partial \theta$  is definitely negative as required by observation.

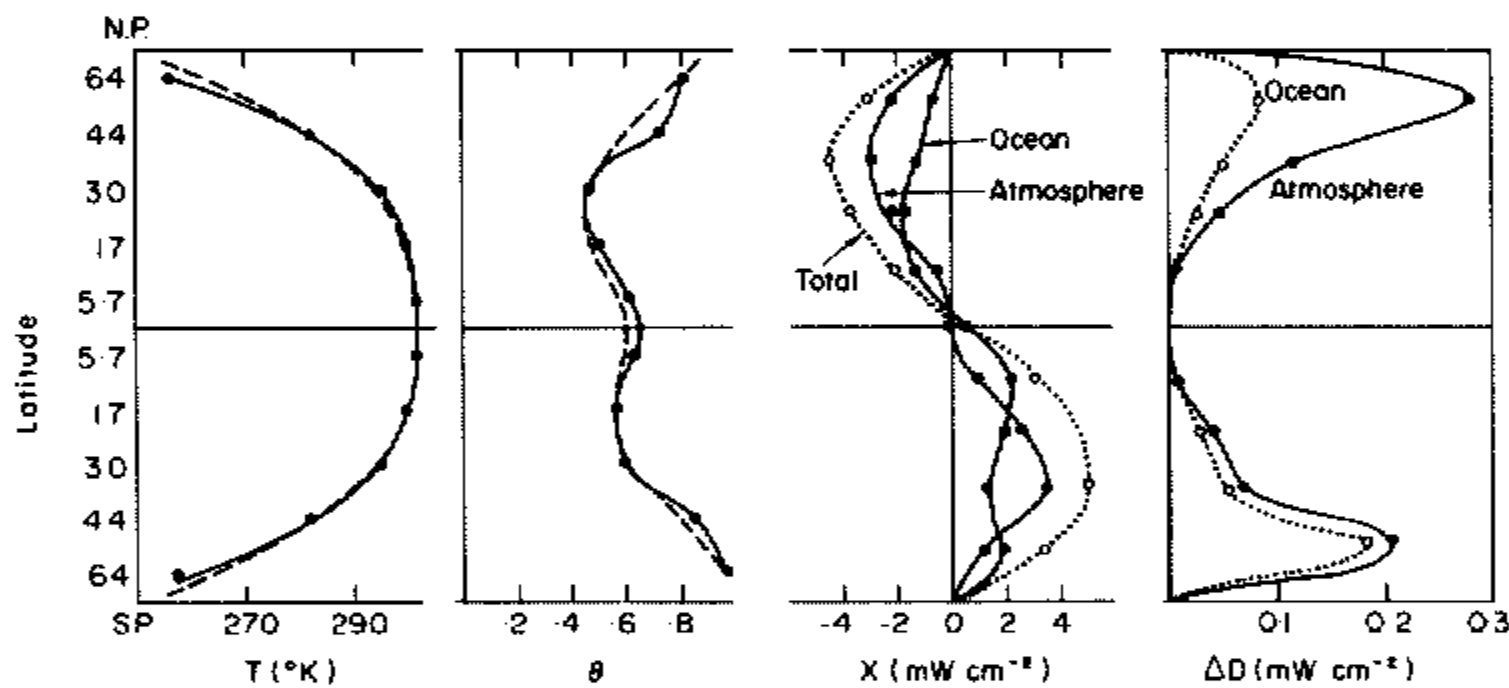


Figure 3. Surface temperature,  $T$ ; cloud cover,  $\theta$ ; oceanic, atmospheric and total meridional energy fluxes,  $X$ ; and zonal oceanic and atmospheric dissipation,  $\Delta D$ . The solid and dotted lines are the curves predicted by the zonal average mean-annual model. The dashed curves are comparative observations (from Crutcher and Meserve 1970; Taljaard *et al.* 1969; those for cloud cover from Landsberg, quoted by Winston 1969). The flux units are normalized by the surface area of the zones (1/10 of global surface area). Thus the difference in flux across a zone is numerically equal to the convergence into a  $1\text{ cm}^2$  vertical column.

Multiplication of the flux by zonal surface area gives the total energy flow across the latitude circle.

The four sections of Fig. 3 are meridional profiles of surface temperature, cloud cover, atmospheric and oceanic meridional energy fluxes, and the zonal atmospheric and oceanic dissipation predicted on the basis of the model. They are the unique distributions which yield a maximum in entropy production for the system as a whole (see Eq. (10)). Arbitrary variation of the starting points of the simplex iteration (i.e. of the initial values of the  $(X_a + X_o)_i$ ) makes no difference – there is no evidence of multiple maxima.

The dashed curves are observed values from sources given in the caption. Agreement with observation is very good. The predicted polar temperatures are a little low, but legitimately could be adjusted by tuning the average values of the constants in those regions. (The extreme zones cover a large range of latitude and solar angle, and incorporate as well the sharp jump in albedo across the ice boundary. An 'average' in such circumstances is difficult to define.) The predicted cloud cover displays the observed equatorial peak and the slightly greater deficit in the northern subtropics as compared with the southern. The meridional profiles of atmospheric ( $X_a$ ) and oceanic ( $X_o$ ) energy flux agree well with what is

known about the northern hemisphere (see, for instance, Palmén and Newton 1969).  $X_o$  dominates at low latitudes and  $X_a$  dominates towards the pole. In the southern hemisphere, where there are no adequate data for comparison, the model predicts a dominance of  $X_o$  at both low and high latitudes. Overall, the ratios of the fluxes and the associated dissipations are determined primarily by the fraction of ocean within each zone, but are modified by the fact that the equator-to-pole temperature gradients are less in the atmosphere than at the surface. This last also agrees with observation.

#### 4. THE THREE-DIMENSIONAL MODEL

If indeed the overall thesis of this paper is correct, it is reasonable to expect that it would work for a simple zonal average model because on the scale of a zonal average the number of allowable dynamic formats (degrees of freedom) might indeed be large. Accepting the necessity for many degrees of freedom to allow the principle to operate, the question arises as to what is the maximum possible resolution or detail in a model such as this before it becomes necessary to include specific consideration of the dynamics. To this end the concept was applied to a world divided into the 400 boxes shown in Fig. 4. Each box has equal surface area ( $\sim 1.27 \times 10^6 \text{ km}^2$ ). The numbers in the boxes are the observed values of surface albedo,  $\alpha$ . They were computed roughly from the mean annual minimum-albedo

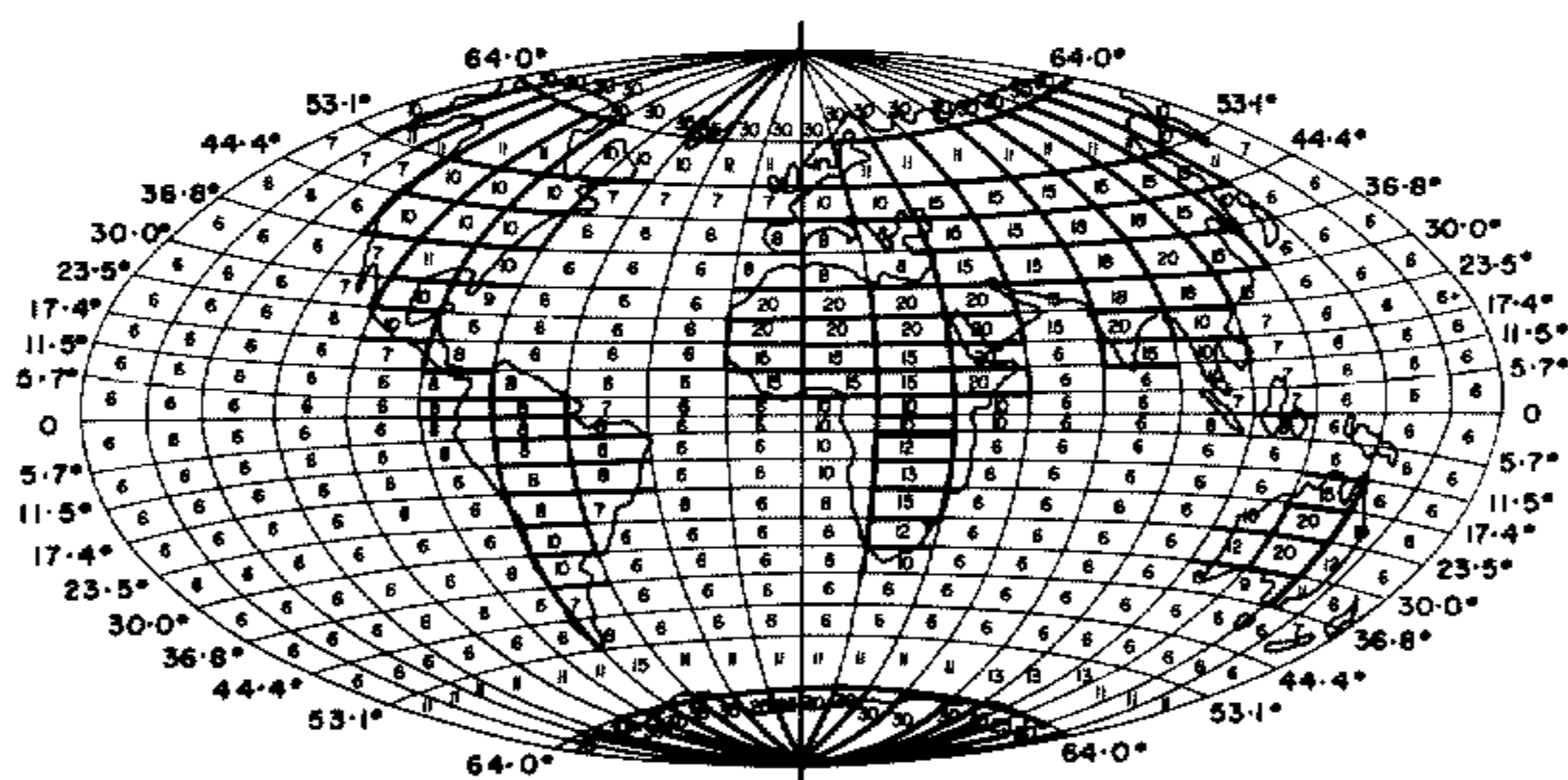


Figure 4. Format of the three-dimension (3-D) model. The numbers in each box are the surface albedos (percent) used as input data. The darkened box edges are those for which no oceanic energy flow is allowed. Latitudes are recorded on the edges of the diagrams.

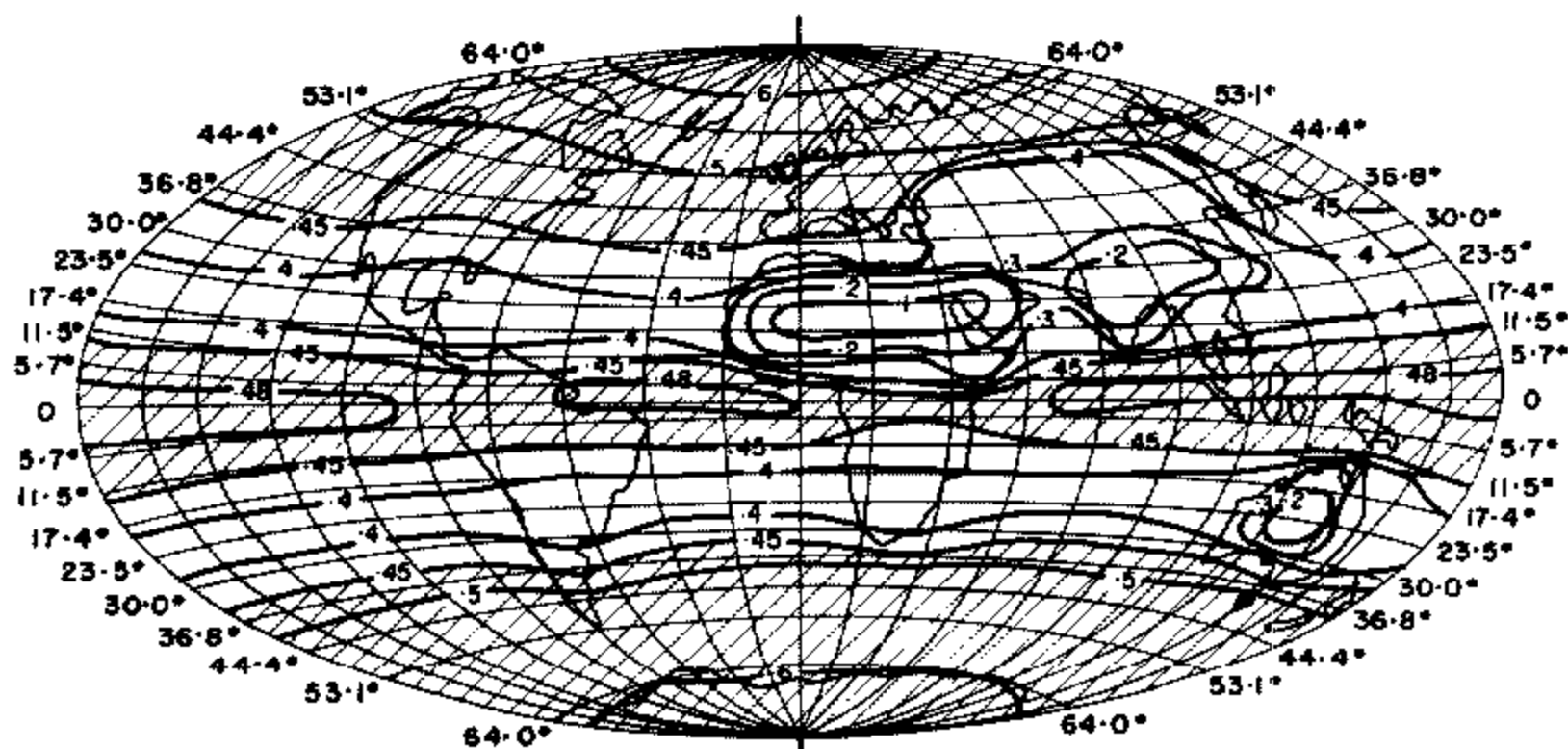


Figure 5 Global distribution of mean annual cloud cover,  $\theta$  (as fraction from 0 to 1), predicted by the 3-D model. Cross-hatching indicates regions where  $\theta > 0.45$ .

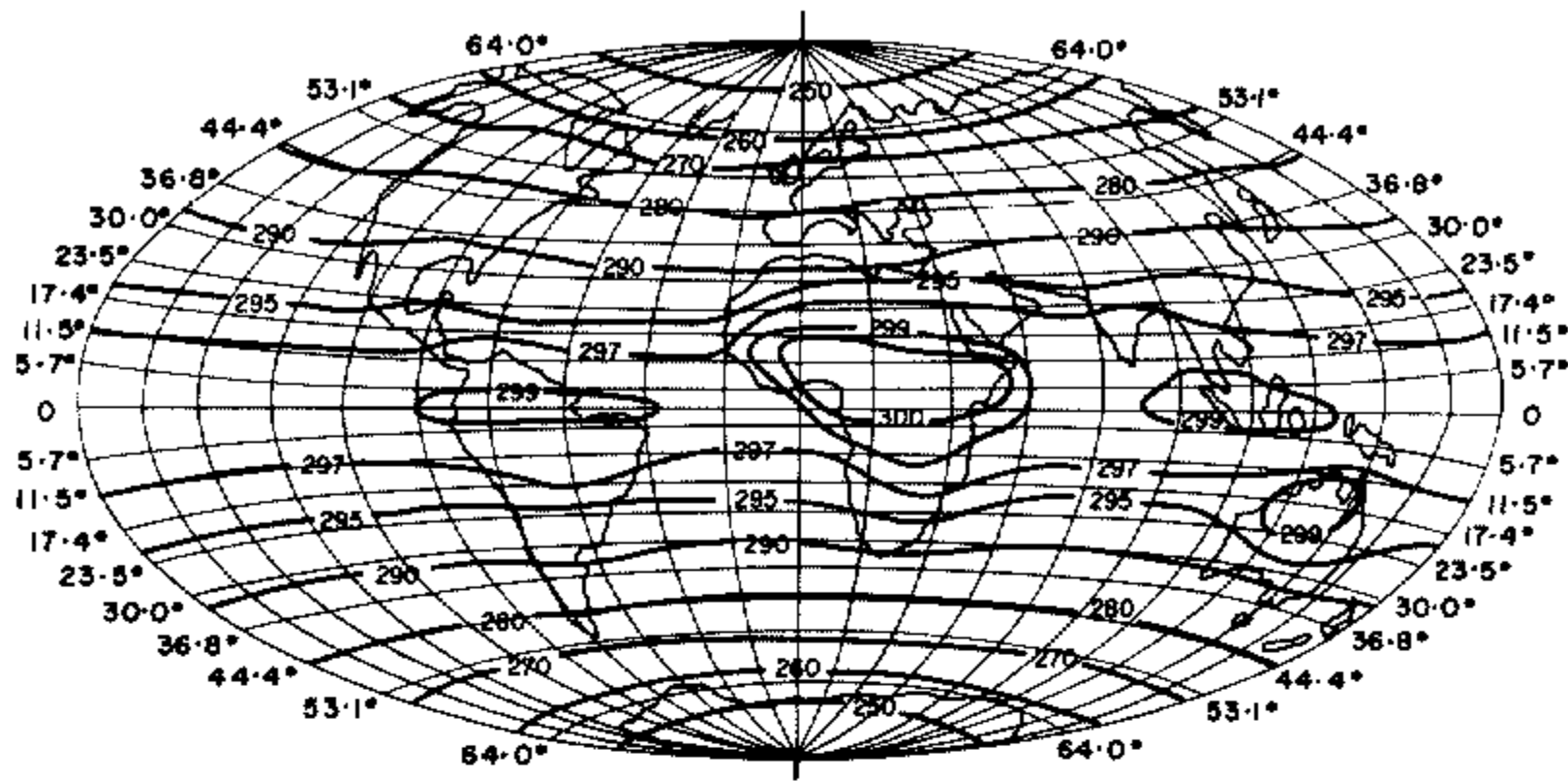


Figure 6. Global distributions of mean annual surface temperature,  $T(K)$  predicted by the 3-D model.

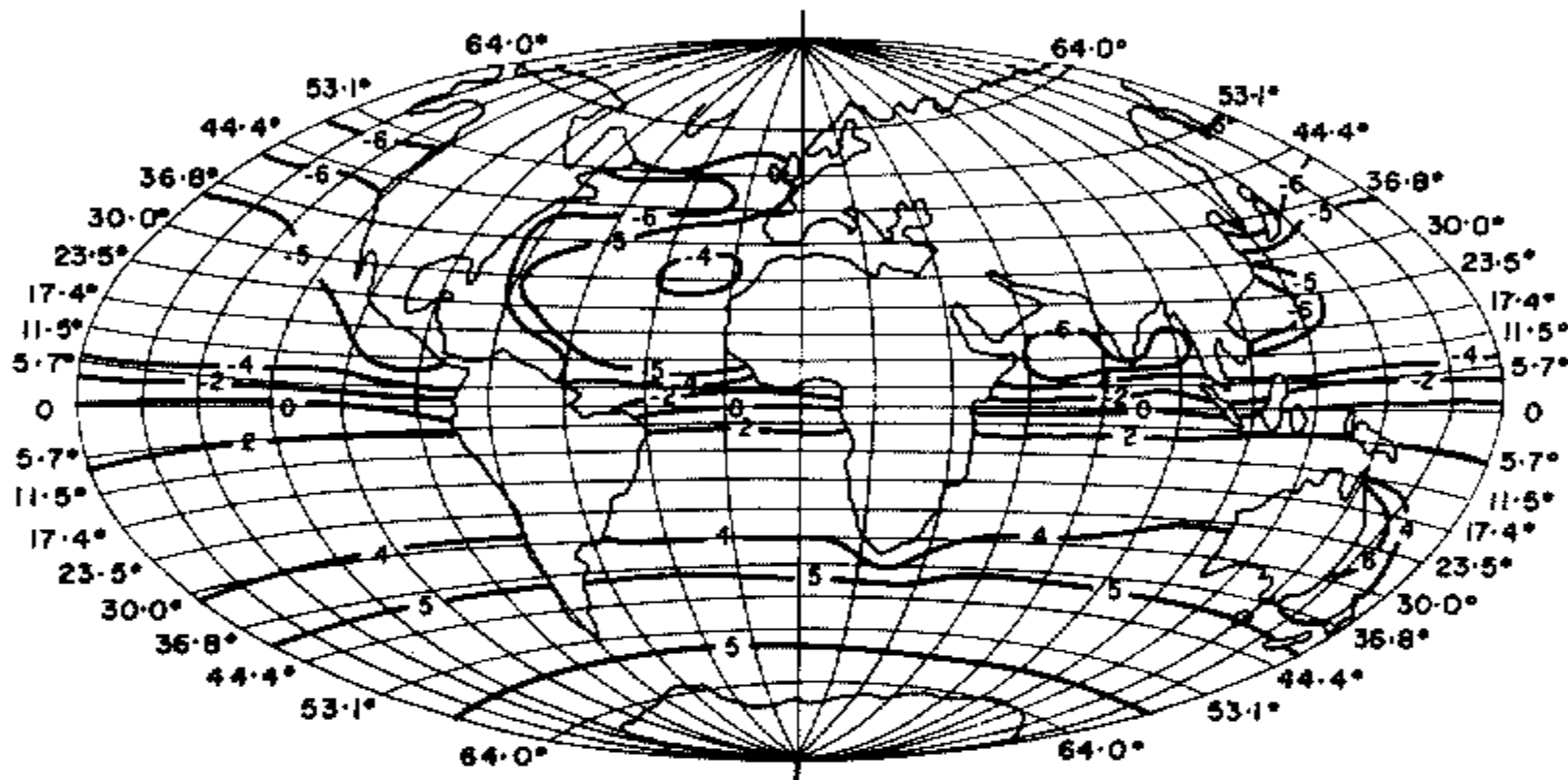


Figure 7. Predicted global distribution of the mean annual north-south component of the oceanic energy flux ( $\text{mW cm}^{-2}$ ). Southward flows are positive. Note that the units refer to a square centimetre of *surface* area in order that the difference in flux across a box is numerically equal to the average convergence into a  $1 \text{ cm}^2$  vertical column.

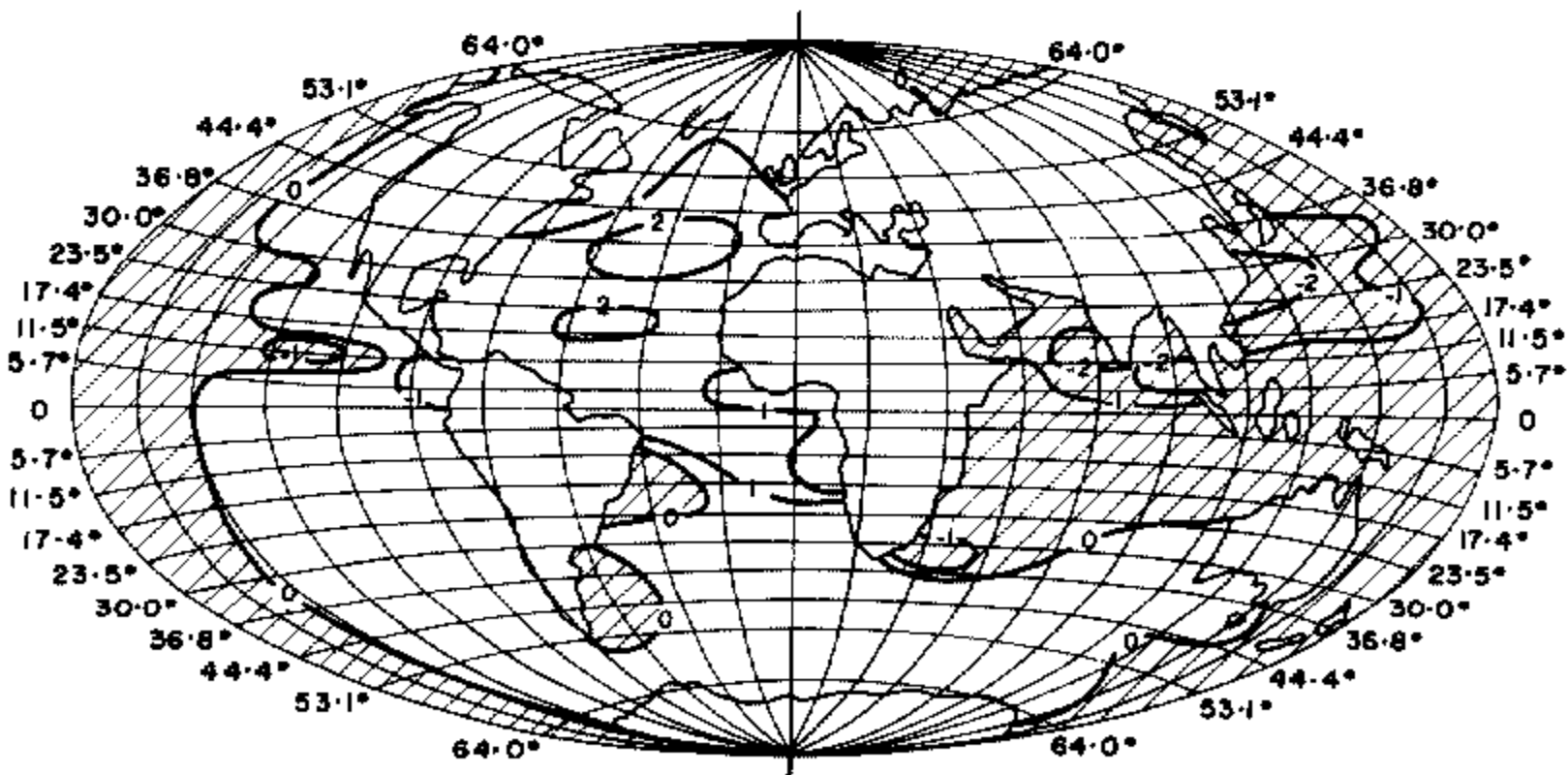


Figure 8. Predicted global distribution of the mean annual east-west component of the oceanic energy flux ( $\text{mW cm}^{-2}$ ). Cross-hatched areas are regions of westward flow. Eastward flows are positive. Note that the units refer to a square centimetre of *surface* area in order that the difference in flux across a box is numerically equal to the average convergence into a  $1 \text{ cm}^2$  vertical column.

satellite data of Raschke *et al.* (1973) with some adjustment to ensure that oceanic areas of the same latitude have the same albedo. The darkened edges of the boxes correspond to those boundaries where horizontal oceanic energy flow was not allowed. The various other

constants were given the same latitudinal dependence as for the zonal average model (see Table 1) using interpolation to obtain the doubled resolution.

The physics of each box was set up in exactly the same way as for the zonal average

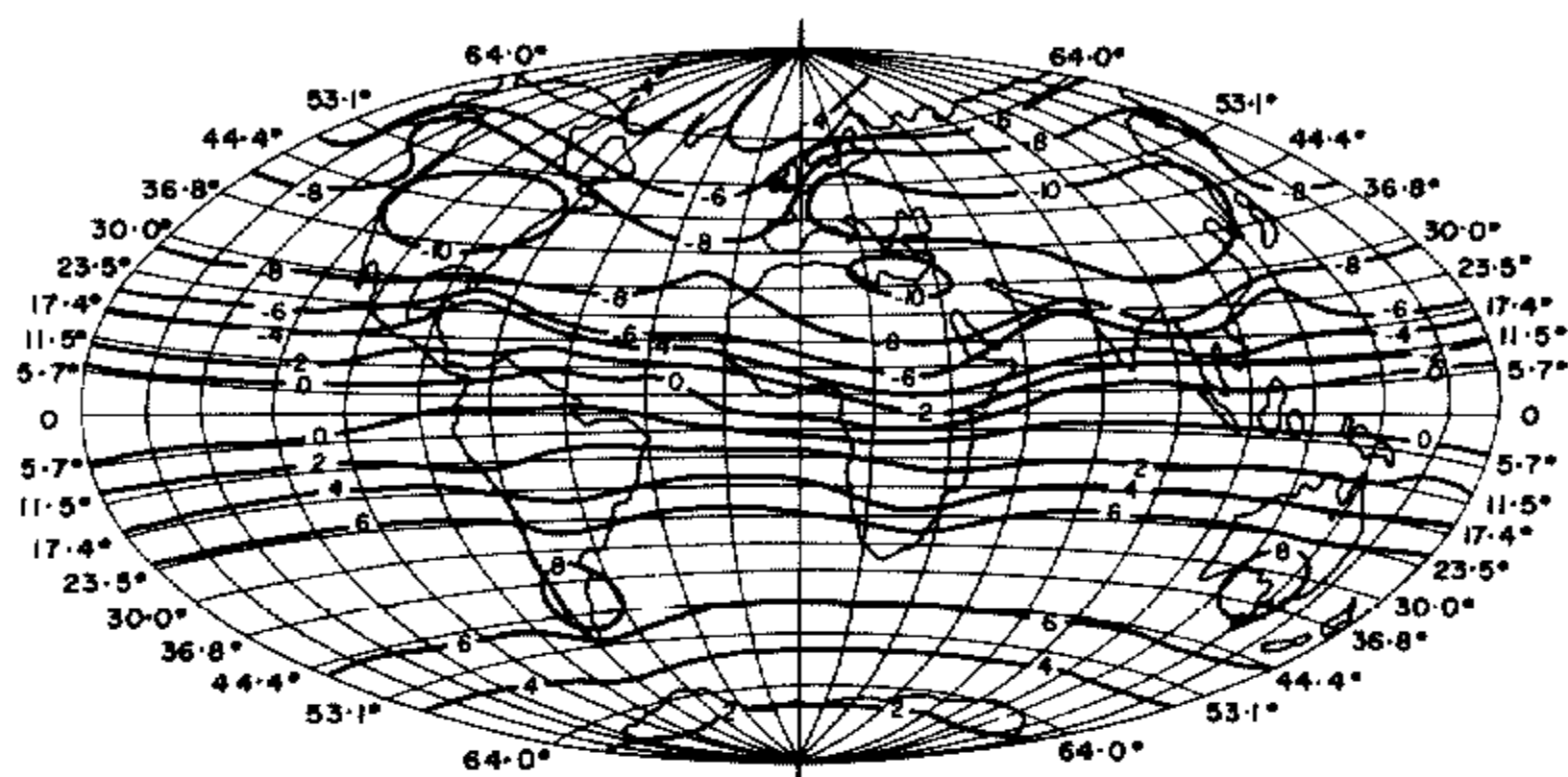


Figure 9. Predicted global distribution of the mean annual north-south component of the atmospheric energy flux ( $\text{mW cm}^{-2}$ ). Southward flows are positive. Note that the units refer to a square centimetre of *surface* area in order that the difference in flux across a box is numerically equal to the average convergence into a  $1 \text{ cm}^2$  vertical column.

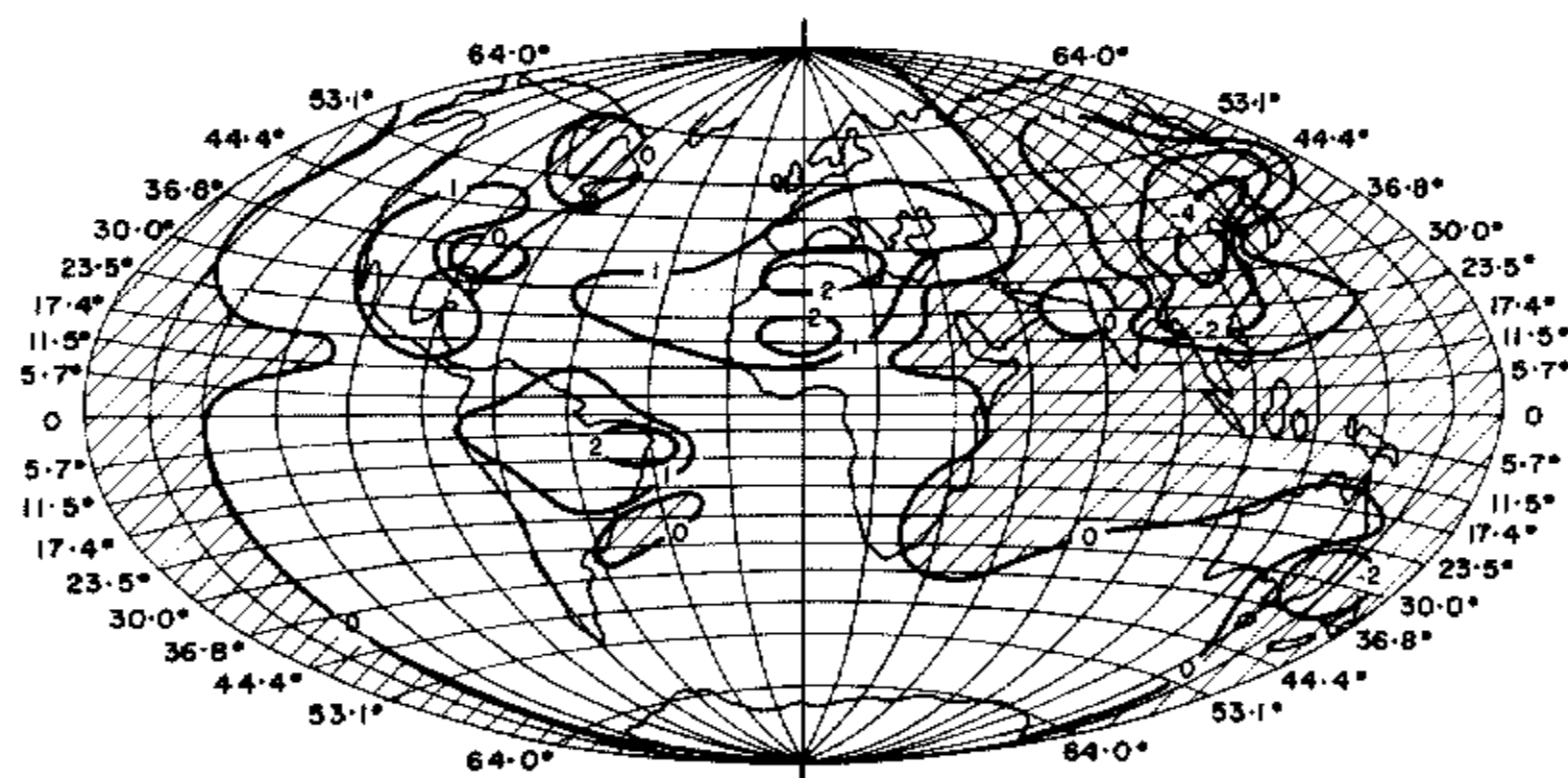


Figure 10. Predicted global distribution of the mean annual east-west component of the atmospheric energy flux ( $\text{mW cm}^{-2}$ ). Cross-hatched areas are regions of westward flow. Eastward flows are positive. Note that the units refer to a square centimetre of *surface* area in order that the difference in flux across a box is numerically equal to the average convergence into a  $1 \text{ cm}^2$  vertical column.

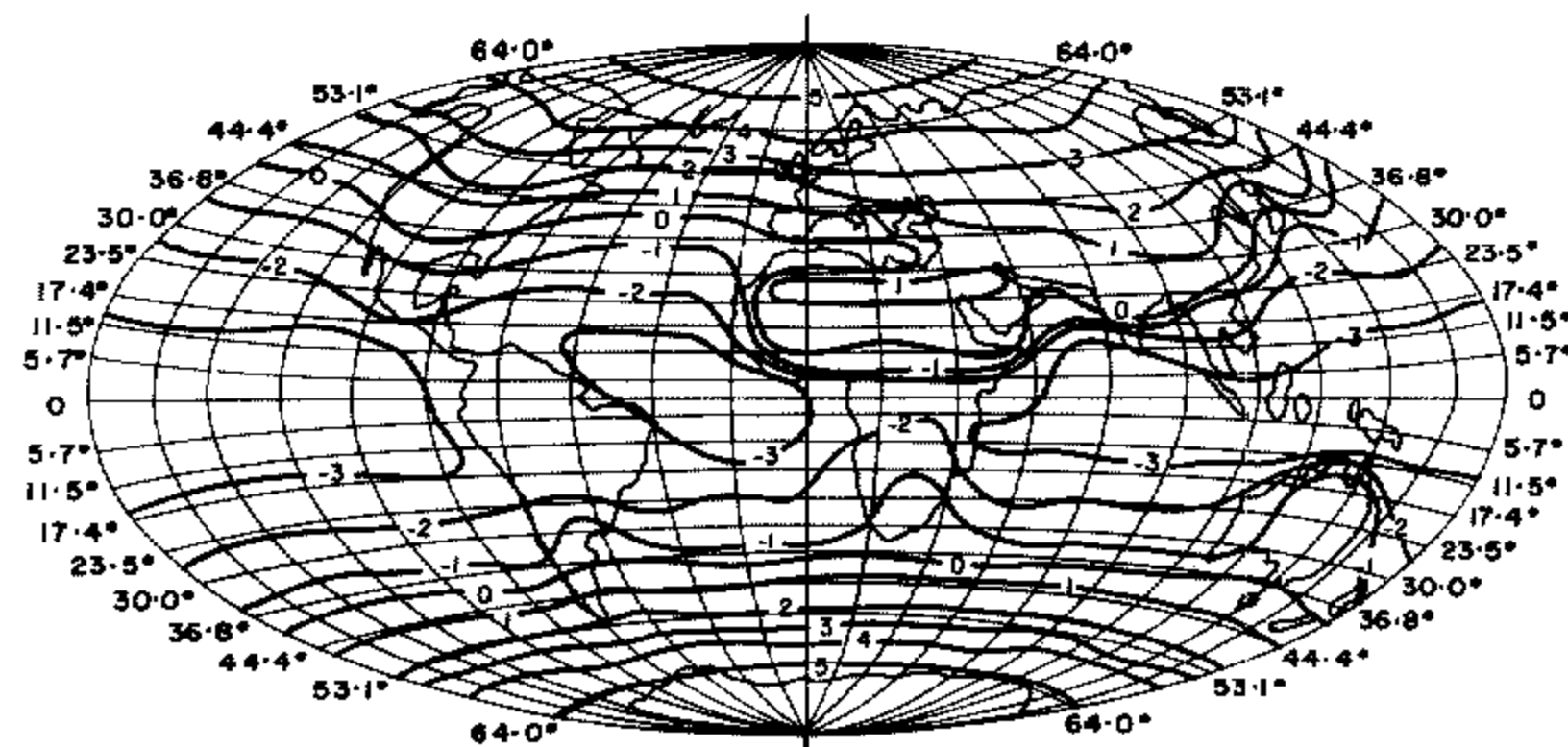


Figure 11. Predicted mean annual global distribution of the total (ocean plus atmosphere) horizontal convergence ( $\text{mW cm}^{-2}$ ). It is also a distribution of the net radiative output from the planet.

case. In that model the maximum in the overall  $\dot{S}_I$  was found with respect to the nine cross-box (meridional) flows of total energy using the elegant numerical technique of the simplex method. In the 3-D model there are 380 cross-box flows to be considered. Precision problems ensure that the simplex method is quite inappropriate. A direct approach was adopted on the basis that no multiple minima appeared in the zonal model. Starting with an arbitrary set of the cross-box flows (e.g. setting them all to zero), the flow between two adjacent boxes was varied and held at the point of maximum entropy production (i.e. minimum entropy exchange) for those two boxes. Each of the cross-box flows was treated in the same manner in a regular pattern. After one complete pass over all the flows the process was repeated until the overall global entropy exchange stopped decreasing. At this point the average entropy production of the globe was  $8.8805 \times 10^{-4} \text{ mW cm}^{-2} \text{ K}^{-1}$ .

Figs. 5 to 11 provide contours of the predicted geographic distribution of cloud, surface temperature, north-south and east-west components of the oceanic energy flux, north-south and east-west components of the atmospheric energy flux and the total convergence of the oceanic and atmospheric energy fluxes. They are mean annual values inasmuch as the model was run with the sun overhead at the equator, and are the steady-state values achieved after 100 passes of the minimization procedure.

There is no question that broad agreement with observation is very good. Relevant data sources include Berry *et al.* (1945), Miller and Feddes (1971), Schutz and Gates (1971-74). In particular, there are features in Fig. 7 which are reminiscent of well-known dynamic phenomena - e.g. the Gulf Stream of the Atlantic, the Kuroshio current to the east of Asia, and the East-Australian current. On the other hand, there is no evidence in the distributions of such things as a slight shift of the intertropical convergence zone to the north of the equator. The east-west oceanic and atmospheric energy fluxes indicate a convergence to the dominant land mass of the Asia-Africa complex. The north-south energy flows in the atmosphere are concentrated over the land.

## 5. DISCUSSION

Referring back to Table 2, there are a number of points concerning system sensitivity.

First, the value of solar constant,  $R_0$ , has no effect on cloud cover,  $\theta$ . Accepting the assumptions of the overall thesis, this is a fortunate circumstance for the many earlier attempts to calculate the response of global temperature and temperature distribution to change in  $R_0$ . Such attempts of necessity involved assumptions of fixed cloud cover. The point has implications also for the modern practice of testing a general circulation model (GCM) in terms of its ability to simulate seasonal variability. Most GCMs to date operate

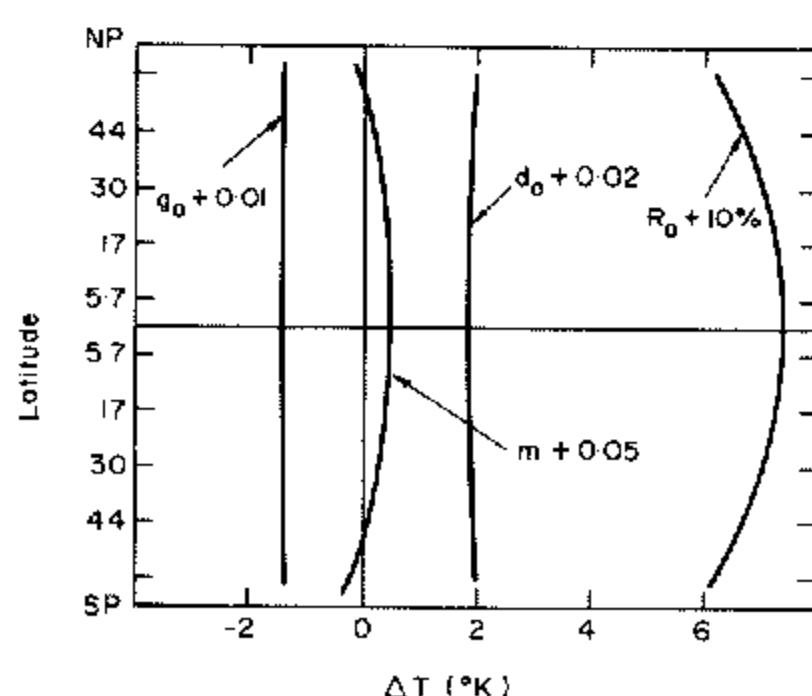


Figure 12. Predicted zonal average changes,  $\Delta T$ , in mean annual surface temperature  $T(\text{K})$  for specified increases in the constants having an effect on solar input. They are derived from the zonal average model, and are the changes from the mean case of Figure 3.

with fixed zonal average cloud. Success in simulation of seasonal variability may not of itself be a valid reason for downgrading the importance of cloud feedback when the concern is with change of other external parameters. The global sensitivity to  $R_0$  predicted here (0.7 K for 1% change in solar constant) is a little lower than many earlier estimates, which range widely about 1.2 K for 1% change (e.g. Manabe and Wetherald 1967). Tests with the zonal average model (see Fig. 12) show that an increase of  $R_0$  also increases slightly the equator-to-pole temperature gradient.

Second, and with the single exception of atmospheric solar absorption,  $m$ , the sensitivity to change of the various parameters is such that a positive response of  $\theta$  is accompanied by a negative response in  $T$  – or vice versa. This is an obvious expectation since an increase in cloud decreases the energy input to the system and one can imagine that both the planet and the surface will cool. However, and again with the exception of the case where  $m$  is changed, increasing cloud is accompanied by an increase in atmospheric temperature,  $T_a$ . This seems peculiar until one remembers that an increase of energy into the atmosphere can be balanced by longwave loss to space from extra cloud through the longwave window. There is here a basic thermodynamic reason for the low cloud amount in the subtropics. These are regions of relatively large divergence of the *atmospheric* meridional energy flux, so that the atmosphere has a relatively small amount of energy to be radiated away. It needs less cloud. The polar regions are subject to meridional flux convergence (mainly atmospheric convergence) and require increased cloud so as to lose the extra energy. In turn, this is related to the smaller equator-to-pole temperature gradient in the atmosphere as compared to the surface.

Third, the sensitivities such as those of Table 2 (which refer only to global mean conditions) are strictly ‘partial derivatives’ when calculated and used for estimating changes in an individual box or zone. Without going into mathematical detail, the best way of describing the situation is to say that the overall maximization process tends to ensure a smooth envelope of boundary (i.e. atmospheric) temperature. A change of box constant which lowers  $T_a$  of that box results in an increased convergence of the horizontal energy flow which tends to restore  $T_a$  to its original value. Thus, as a very approximate rule of thumb, the full derivative effect on  $\theta$  of a change in any parameter  $p$  can be assessed somewhat awkwardly as follows:

$$d\theta/dp \approx \delta\theta/\delta p + (\delta\theta/\delta(\Delta X))(\bar{\delta}(\Delta X)/\delta p) \quad (13)$$

where  $\delta\theta/\delta p$  and  $\delta\theta/\delta(\Delta X)$  are sensitivities such as those quoted in Table 2. At constant convergence  $\Delta X$ , a change  $\delta p$  gives a change in  $T_a$  equal to  $(\delta T_a/\delta p)\delta p$ . The overall maximization process ensures that  $\Delta X$  changes by an amount  $\bar{\delta}(\Delta X)$  such that

$$(\delta T_a/\delta(\Delta X))\bar{\delta}(\Delta X) \approx -(\delta T_a/\delta p)\delta p \quad (14)$$

from which

$$\bar{\delta}(\Delta X)/\delta p \approx -\frac{\delta T_a/\delta p}{\delta T_a/\delta(\Delta X)} \quad (15)$$

The right hand side of Eq. (15) consists again of sensitivities such as those in Table 2, so that the  $d\theta/dp$  of Eq. (13) can be calculated. Similar expressions apply when  $\theta$  is replaced by  $T$ . Consider the mythical zone most representative of global average conditions so that the data of Table 2 are appropriate. The direct responses of  $\theta$ ,  $T$  and  $T_a$  to increased convergence are all positive.  $\delta\theta/\delta p$  and  $\delta T_a/\delta p$  are always of the same sign while  $\delta T/\delta p$  and  $\delta T_a/\delta p$  are always of opposite sign. Thus the sensitivity of  $\theta$  to any change in box constant is reduced well below that indicated by the ‘partial derivative’ (and sometimes to the extent of sign reversal) while the sensitivity of  $T$  is amplified beyond the ‘partial derivative’ expectation.

The exception of atmospheric shortwave absorption ( $m$ ) is very important. It arises because of the direct association between planetary albedo and solar absorption (Eqs. (9)).



Without the association, an increase in  $m$  has no effect on  $\theta$ ,  $T$  or  $T_a$ . Without the association, an increase in the planetary albedo of clear skies is accompanied by large increases in cloud cover and large decreases in surface temperature – an untenable situation over desert areas. Thus in the present model the deficit of  $\theta$  and slight increase in  $T$  over desert areas does not arise because of low moisture content in the atmosphere. It arises because there is not a one-to-one relation between solar energy into the planet and solar energy into the ground when the surface albedo is changed.

The impressive 'reality' of the results of the 3-D model owes much to this effect. Increased surface albedo of a box increases the surface temperature and decreases the cloud and atmospheric temperature in comparison with other regions at the same latitude.

Regarding the actual numerical manipulations, it is dangerous to impose artificial constraints to ensure the minimization procedure does not stray through physically impossible situations of (say) negative  $\theta$ . They tend to act as false boundary conditions and lead to false results. They are not necessary for the mean annual cases reported here, and the initial values of the cross-box flows can be set over a wide range including zero. Some care must be exercised for extreme season simulations. For these the initial values of the polar region fluxes must be reasonably correct.

Finally it should be mentioned that the original constraint of I (that of minimizing entropy exchange defined as  $\sum_{i=1}^{10} (F_{\downarrow s} - F_{\uparrow L}) / (F_{\uparrow L})^{\frac{1}{2}}$ ) does not work for the present model. Bearing in mind the out-of-phase relation between  $T$  and  $T_a$ , it can be seen in any event that a definition of boundary temperature as proportional to  $(F_{\uparrow L})^{\frac{1}{2}}$  is not appropriate. Such a definition ensures purely one-dimensional behaviour since there is only the one concept of temperature – a sort of average of the surface and the atmosphere. For this reason it was impossible in I to separate the atmospheric and oceanic contributions to meridional energy flow. For this reason also, the artificial and (apparently) two-dimensional physics of the model of I still allowed a correct one-dimensional answer (see Rodgers 1976).

## 6. CONCLUSION

The basic thesis of the paper is that there are many steady-state formats of the earth-atmosphere system which could satisfy the boundary conditions; and it is assumed by analogy with certain small-scale turbulent transfer processes that the observed format is one which satisfies a thermodynamic principle of maximum entropy production. The concept is similar to that of Lorenz, who proposed selection of the mode of maximum efficiency. In order to test the idea, the principle has been applied to a global model containing no restrictions on the dynamics – it was assumed that there are an infinite number of possible formats.

Despite the obvious objections which can be made to the last assumption, there can be no question that the predicted format of such things as cloud cover or surface temperature is very realistic on the broad scale of, roughly, several thousand kilometres. The results could be made even better by deliberate (and legitimate) tuning or parameterization of the various thermodynamic constants of the individual regions.

The apparent irrelevance of dynamics to the large-scale thermodynamics is the point most difficult to believe. The model suggests, for instance, that changing the earth's rotation rate would have no effect on the broad thermodynamics. Two comments can be made. First, the various theoretical, numerical and experimental studies which emphasize the relation between dynamics and thermodynamic structure generally concern systems of very restricted degrees of freedom, and of artificially constrained boundary conditions. Rotating tank experiments are a good example. They normally involve energy flow in one medium via a

single turbulent transfer mechanism. The boundary temperatures are normally fixed. The earth-atmosphere has two media, a number of possible modes of energy transfer in each, and unconstrained thermodynamic boundary conditions in that the internal dynamics can feed back on the energy inputs via modulation of such things as cloud cover. Second, the 'large-scale' here concerns both space and time. The model deals only with the multi-annual mean situation, for which an irrelevance of dynamics to the large-scale thermodynamics is at least more plausible than for shorter time scales.

Nevertheless it is certainly probable that very large changes in the dynamic constraints would have an effect. Large changes might shift or restrict the system so that none of the allowed dynamic formats could achieve the optimum thermodynamic structure.

Two other concepts are built into the model. The first is the maximization of the vertical ocean-to-atmosphere convective heat flow ( $LE + H$ ) within the two constraints set by energy balance of the surface and of the atmosphere. This is simply an extension of the overall thesis and is presumably related to the observations and suggestion of Malkus and Veronis of a principle of maximum convective heat transport. The second is a result which derives from acceptance of the overall thesis and the behaviour of the model. It is that the partition of the total horizontal energy flow between the atmosphere and the ocean is governed by a tendency to equalize the dissipations of the two media.

Finally, there is no suggestion here that individual phenomena are not of dynamic origin – or that they cannot be explained by purely dynamic arguments. The thermodynamic concept may 'explain' the broad-scale energy flows, but dynamic arguments are necessary to understand how those flows are achieved.

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