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Global dynamics and climate – a system of minimum entropy exchange

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SUMMARY

It is found that the mean meridional distribution of temperature, cloud cover and meridional energy flux can be predicted with extraordinary accuracy by application of a simple minimum principle to a multi-box model of the globe which contains no direct specification of the system dynamics. The minimized quantity is related to the global net rate of production of entropy. It is the sum over all latitude zones of the ratio of net radiant energy input to the effective emission temperature of the zone. The result suggests that global dynamics is something of a passive variable which alters so as to satisfy a condition akin to minimum energy dissipation.

1. INTRODUCTION

The motions of the atmosphere and ocean obey basic physical laws such as the conservation of mass, energy and momentum. However, these laws have not proved sufficient for closure of the problem of atmosphere–ocean dynamics. It is not yet possible to predict *a priori* either the global mean climate or the global distribution of climate. Further, it is not possible to calculate the change in climate which might occur as a result of a change in any of the external system parameters such as the energy output of the sun.

Research on the problem is intense. The modern approach assumes that a solution will be possible when all significant processes affecting weather and climate can be described in sufficient detail. Vast numerical models have been developed which attempt to simulate global dynamics and climate at the maximum level of detail compatible with available computers (see for instance GARP Report 14 1973). The work is urgent and necessary, but it can be argued that the approach is at variance with a basic philosophy of physics which, when faced with a complex problem, searches for simple laws which may govern the overall situation. Ideally, such laws should be simple in both concept and application.

This paper reports the discovery that global dynamics and climate may be ' controlled ' by a simple minimum principle where the minimized quantity is related to the global rate of production of entropy. Application of the principle allows *a priori* prediction of the zonal mean temperature, cloud cover and meridional energy flow over the entire globe *without recourse to specific description of atmospheric and oceanic dynamics*. It may therefore be a powerful boundary condition for many of these dynamical problems. Further, the principle allows calculation of climate change on the basis that the dynamics of the system is something of a passive variable which alters appropriately so as to satisfy the criterion of minimum entropy production.

The work began with the hypothesis that there are sufficient degrees of freedom in the complexity of global dynamics for control by some minimum principle to be possible. There are many such ' laws ' in physics. One characteristic they seem to have in common is that there is no way of telling beforehand which quantity is to be made a minimum (Yourgrau and Mandelstam 1960). Here, in order to search for such a law, it was necessary first to develop a model of an atmosphere–ocean 'box ' which was representative of the mean conditions of a latitude zone. It had to be internally mechanistic in the sense that only the net meridional energy flows into the box need be specified for the model itself to calculate the basic quantities of surface temperature T and cloud cover θ. Having devised such a model, an *overall* multibox model of the world could be assembled where the inter-box (i.e. meridional) flows of energy could be varied arbitrarily. The problem then reduced to a search for any single parameter of the overall system which was minimized for a unique set of the meridional flows – a set, furthermore, which was close to that observed on the real world and which led to the observed distribution of cloud cover and temperature. It was assumed that if an overall minimum principle is operative it should be sufficiently powerful to be apparent despite the inadequacies of the model employed in its search.

Initial evidence for the existence of such a principle was obtained with a three-box model of the mean annual hemisphere. It was found that a unique set of meridional energy flows close to those observed yielded a minimum in the quantity $\sum_i (F_{Si}/F_{Li})$ – that is, in the sum over the three boxes of the ratio of absorbed solar energy F_S to the infrared or long-wave energy F_L radiated to space. The matter was pursued by examining the properties of a ten-box model of the entire globe. Results of this examination are reported here.

2. The 'individual box' model

The model devised is superficially similar to that reported earlier by Paltridge (1974), but is in fact quite different from the earlier attempt. It is based on energy balance requirements and consists of the formulation of three independent balance equations containing only three unknowns (cloud cover, θ, surface temperature, T, and $LE + H$ – see below) apart from the meridional energy flows.

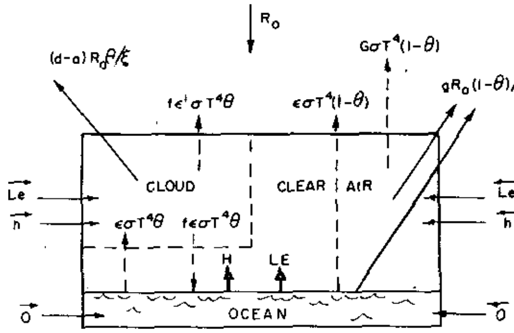

Figure 1. Schematic presentation of the energy flows associated with an individual latitude zone.

Referring to Fig. 1, energy balance of the ocean within a latitude zone requires that

$$LE + H = \{[R_0 - gR_0(1 - \theta) - dR_0\theta - mR_0]/\xi\} - \varepsilon\sigma T^4 + f\varepsilon\sigma T^4\theta + O_N \quad . \qquad (1)$$

where LE and H are the upward surface fluxes of latent and sensible heat into the atmosphere. The first term on the right-hand side is the net short-wave flux into the surface. Within this term R_0 is the solar constant, g the planetary albedo of the cloud-free portions of the zone, and d is the sum of the planetary albedo and the extra short-wave absorption by liquid water of those portions which are cloud covered. The gases of the atmosphere absorb a fraction m of the solar input, and ξ is the ratio of the actual surface area to the projected area as seen by the sun. The second and third terms on the right-hand side represent the net long-wave flux into the surface (the upward and downward fluxes respectively); σT^4 is the black-body radiation at surface temperature T (Stefan-Boltzmann law); ε is a fraction related to the width of the window region of the black-body spectrum ($\approx 7{\cdot}5$–$12{\cdot}5\mu$m) where, for surface conditions, dominant long-wave exchange takes place; and f is the fraction by which black-body radiation from the clouds is reduced below surface black-body emission by reason of their lower temperature. All the above are discussed more fully in the earlier paper. The last term, O_N, is the net meridional oceanic energy flow into the zone.

Similarly, energy balance of the atmosphere within a latitude zone requires that

$$LE + H + R_0(m + a\theta)/\xi + (1 - f)\varepsilon\sigma T^4\theta + Le + h = \varepsilon' f\sigma T^4\theta + G\sigma T^4(1 - \theta) \quad . \qquad (2)$$

The energy inputs are on the left-hand side. The third term includes the factor a which takes explicit account of the extra short-wave absorption by the liquid water in clouds. The term $(1-f)\varepsilon\sigma T^4\theta$ is the net long-wave absorption in clouds resulting from exchange with the earth's surface. Le and h are the net meridional flows of latent and ' sensible ' heats into the zone. On the right-hand side the first term is the long-wave radiant energy lost to space by clouds. The last term is the long-wave loss to space by the clear-sky portions of the zone. G is a constant based on the empirical observation that the long-wave loss to space by average clear skies is closely proportional to the long-wave loss by the underlying surface. ε' is a constant related to the width of the window region of the infrared spectrum above the bulk of atmospheric water vapour. Both constants are discussed in the earlier paper.

It was pointed out in that work that latent heat release occurs in cloud. Sensible heat from the surface is expended mainly on raising the temperature of the lowest levels of the atmosphere. The upper and lower regions so defined tend to be separated by a stable layer across which there is little sensible heat exchange. In the broadest sense this layer can be visualized at or about the height of average cloud base, but it exists under both cloudy and non-cloudy conditions. During cloudy periods there is little long-wave radiative loss from the lower region. In fact, provided the stable layer is persistent and provided there is no significant meridional flow of sensible heat in these lowest atmospheric levels, energy loss from the region occurs only during clear skies. Thus we have a further energy balance relation

$$H = kG\sigma T^4(1 - \theta) \quad . \quad . \quad . \qquad (3)$$

where k is that fraction of radiation lost from the total clear-sky atmosphere which derives from below the stable layer. In order to evaluate k we make use of an empirical observation and a broad principle. First, the loss of long-wave energy from any pressure height of zonal-average clear skies decreases almost linearly with pressure. Thus the fraction of the atmospheric total of long-wave radiation lost to space by a given pressure interval is proportional to the size of that interval. Second, the pressure height to which a parcel of air will rise is roughly proportional to its excess of temperature or heat content over that of the surroundings.

With this in mind, visualize the flux H in terms of buoyant eddies rising from the surface. Imagine also that the cloudy region consists of buoyant parcels heated by the release of latent heat. In both cases the pressure height to which the parcels rise will be set by their heat content. For the cloudy region the maximum possible heat content (or heat which can be released) is set by the maximum possible total water content of the parcels – i.e. surface specific humidity.

The successful theories which predict the Bowen ratio, H/LE, as a function of temperature (e.g. Priestley and Taylor 1972) are based on the concept that the fluxes are proportional to their individual source strengths. Whatever is the source strength of the flux H (or the initial heat content of the eddies associated with H), it is assumed that the source strength of the flux LE is proportional to the surface specific humidity. Therefore if the foregoing visualization is correct in broad principle, the ratio of the pressure thicknesses of the lower and upper regions of the atmosphere should be roughly proportional to H/LE. Thus the k of Eq. (3) can be replaced by $k_1 H/(LE + H)$ to yield

$$LE + H \approx k_1 G\sigma T^4(1 - \theta) \qquad (3a)$$

The value of k_1 was chosen to satisfy *total* global mean conditions and has the value 1·8. Qualitatively it should be greater than unity since rising air parcels in the upper atmosphere are working against a more positive potential temperature gradient. Since also the ratio $k_1 H/(LE + H)$ is relatively small for most latitude zones (i.e. the pressure thickness of the lower region is relatively small) it is assumed that the meridional energy flows LE and H are confined primarily to the upper region.

Eq. (1), (2) and (3a) describe a system containing only three unknowns θ, T and $LE + H$. They can be solved to yield θ and T for any specified values of the meridional energy flows $LE + H$ (atmospheric flux) and O_N (oceanic flux). As will be seen later, on the broad scale of zonal averages the model produces remarkably realistic values of cloud cover and surface temperature when supplied with realistic values of meridional energy flux. It has two properties of general interest. First, cloud cover and temperature appear simply as passive variables which alter so as to satisfy energy balance. Thus, for example, the amount of cloud need bear no relation to the supply of water vapour – as is obvious anyway by comparing the cloud covers of the polar and subtropical regions. Second, there is no requirement to evaluate the Bowen ratio either explicitly or implicitly. This is in direct contrast to the requirements of the more detailed dynamical models.

3. Fixed parameter values

Table 1 gives the various ' constants ' used in the present work. The planetary albedo of clouds, $(d-a)$, was assumed to have the zenith angle dependence indicated by the theoretical calculations of Fritz (1954) and to have a global mean of 0·45 (see, for instance, Houghton 1954). In the absence of detailed information on cloud droplet absorption a, this factor was set at a constant 0·04 (Fritz 1958). Absorption by atmospheric gases was assumed to have the zenith angle dependence indicated by Yamamoto (1962) and by Paltridge (1973) on the basis of a mean atmospheric water content of 2cm. The ξ are numerical values of the geometrical quantity $\pi/\cos\phi$ where ϕ is the latitude angle. The only ' non-symmetrical ' constant is the planetary albedo g of the clear sky portions of the globe. The planetary albedos of average sea g_S and average land g_L were given the zenith angle dependence shown in the table (see for instance Paltridge and Sargent 1971) and g was then computed as the mean of the two, weighted by the appropriate land fraction for that zone. The other parameters have the values discussed in Paltridge (1974).

TABLE 1. Zonal average constants used in the present work

Mean Latitude (°)	64N	44N	30N	17N	6N	6S	17S	30S	44S	64S
d	0·55	0·50	0·45	0·43	0·41	0·41	0·43	0·45	0·50	0·55
m	0·20	0·18	0·17	0·16	0·16	0·16	0·16	0·17	0·18	0·20
g	0·22	0·18	0·16	0·15	0·14	0·14	0·14	0·14	0·14	0·16
a	0·04	0·04	0·04	0·04	0·04	0·04	0·04	0·04	0·04	0·04
ζ	7·17	4·40	3·63	3·29	3·14	3·14	3·29	3·63	4·40	7·17
Land fraction	0·70	0·50	0·40	0·25	0·20	0·22	0·2	0·17	0·03	0·05
g_L	0·24	0·21	0·20	0·20	0·20	0·20	0·20	0·20	0·21	0·24
g_S	0·16	0·14	0·13	0·13	0·13	0·13	0·13	0·13	0·14	0·16
Atmospheric flux / Oceanic flux	6·6	3·1	1·2	0·57	0·29	0·62	1·6	3·5	7·3	

$R_0 = 136 \text{mW cm}^{-2}$: $\varepsilon = 0{\cdot}30$: $\varepsilon' = 0{\cdot}75$: $f = 0{\cdot}30$: $G = 0{\cdot}31$

4. The overall global model

The world was divided into 10 latitude zones of equal surface area as depicted in Fig. 2. The latitude limits and the mean latitude of each zone are given on the figure. There are 18 inter-box (meridional) energy flows to be considered and they are the ' unknowns ' of the problem. Provided they can be specified, the cloud cover, θ_i, and the temperature, T_i, of each zone can be calculated using the model of an individual box. Derived quantities such as the absorbed solar energy, F_{Si}, and long-wave energy radiated to space, F_{Li}, can also be calculated.

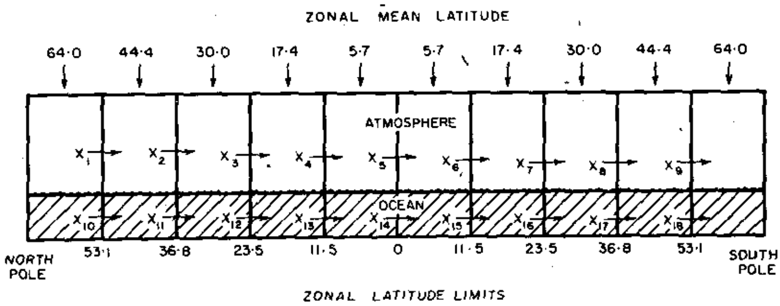

Figure 2. Division of the world into 10 latitude zones or boxes.

An overall quantity such as $\sum_i (F_{Si}/F_{Li})$ ($= Q$ say) is a function of all the interbox energy flows X_i and can be calculated by computer for any set of X_i. There are several numerical techniques for finding the minimum (or minima) of such a function. Here, the simplex method of Nelder and Mead (1965) was used.

As in the case of the three-box model it was found that no absolute minimum exists in the quantity Q without application of an additional physical constraint. It was therefore assumed that at any given latitude the ratio of oceanic to atmospheric flux is a constant – since they are driven by approximately similar (and, in the model, identical) temperature gradients. Such an assumption is easy to justify in cases of pure Fickian diffusion but is highly questionable when applied to non-isotropic flows. However, the assumption does not *require* isotropic diffusion. A sufficient condition is for instance that the total oceanic and total atmospheric meridional flows of energy at a latitude are individually proportional to the temperature gradient.

The observed ratios of atmospheric to oceanic flux (from Sellers, quoted by Palmén and Newton 1969) are given in Table 1. Application of this additional constraint reduced the ' unknowns ' from 18 to 9, and revealed an absolute minimum in Q at values of total meridional energy flow very close to those computed from experiment.

The problem with Q is one of physical interpretation. Therefore at this point the properties of the parameter $\sum_i \{(F_{Si} - F_{Li})/T_{ei}\}$ were examined, where T_e is an effective temperature closely related to F_L via the Stefan Boltzmann law $F_L = \sigma T_e^4$. (In comparison, note that a minimum in Q will occur at the same point as in $\sum_i (F_{Si}/F_{Li} - 1)$ and therefore in $\sum_i \{(F_{Si} - F_{Li})/F_{Li}\}$.) This new quantity ($= E$ say) can be formally interpreted as a rate of entropy production of the system and is intuitively more likely to be a relevant parameter.

Figs. 3(a) and (b) give the latitudinal profiles of surface temperature and cloud cover predicted on the basis of the unique set of meridional energy flux profiles (Fig. 3(c)) corresponding to a minimum in E. The observed profiles are given as the dashed lines in each figure. The measured temperatures are from Crutcher and Meserve (1970) for the

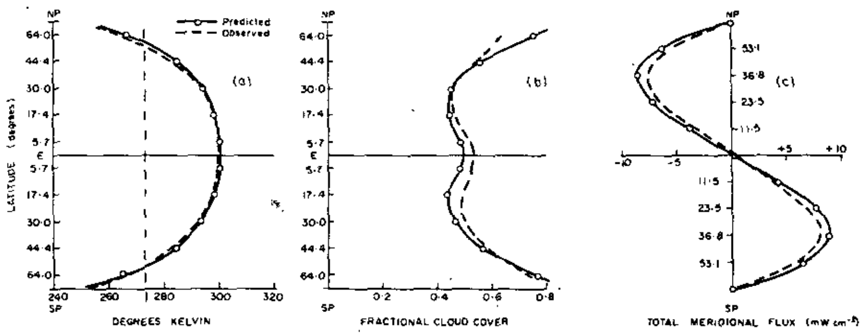

Figure 3 (a), (b), and (c). Predicted and observed meridional profiles of surface temperature, cloud cover and meridional energy flux. The fluxes are normalized to unit surface area of a zone.

northern hemisphere and from Taljaard *et al.* (1969) for the southern hemisphere; the measured cloud covers are from Landsberg (quoted by Winston 1969); and the measured meridional fluxes are from Sellers. Agreement in all cases is quite remarkable. It is in fact better than the agreement with the profiles calculated on the basis of minimizing Q, which tends to produce temperatures toward the poles which are slightly low.

The predicted temperature profile of the northern hemisphere is almost a direct mirror of that for the southern hemisphere. This is shown also by the observations, despite the widely held belief that the northern hemisphere is the warmer by 2 or 3 degrees (e.g. Haurwitz and Austin 1944). (In this connection, the commonly quoted temperatures have been ' reduced ' to sea level. Here we are concerned with ambient surface conditions.) The similarity between hemispheres occurs in spite of the much higher albedo north of the equator. It is encouraging that the observed characteristic is so well reproduced. Furthermore the model produces a world in which the net radiation budget of each hemisphere separately is very close to radiative balance with little energy exchange across the equator. This result, although somewhat at variance with earlier work, is supported by the most recent and authoritative satellite measurements of Vonder Haar and Suomi (1971).

5. Discussion

A formal definition of the rate of entropy production, dS/dt, of the earth-atmosphere system requires the microscale specification of all energy flows or energy transformations together with their associated temperatures. Then $dS/dt = dS_e/dt + \sum_j dS_j/dt$, where dS_e/dt describes external entropy exchanged between the system and its surroundings and the dS_j/dt are the production rates of all the irreversible processes occurring within the system.

In terms of the present model the quantity E is an approximation to dS_e/dt. It is an approximation because it deals with macroscale averages. Furthermore, the assumption that T_e is exactly proportional to $F_L^{\frac{1}{4}}$ need not be true since the spectrum of outgoing radiation is not that of a uniform blackbody. The various wavelengths derive from different atmospheric levels at different temperatures. Thus it may be that T_e should be defined as some greater power of F_L. (The difference in position of the minimum would be very small – as is indicated by the earlier comparison with the parameter Q.)

Assuming that the macroscale approximation is good, the present results imply that the irreversible processes within the system produce entropy at rates which are either insignificant, constant, or directly proportional to E.

Consider the entropy production rate dS_1/dt associated with the total meridional energy flows x_i. (Referring to Fig. 2; $x_1 = X_1 + X_{10}$, $x_2 = X_2 + X_{11}$ and so on.) Again *in terms of the present model a quantity E_1 can be defined which is a macroscale approximation to dS_1/dt.* That is $E_1 = \sum_i \{(x_i - x_{i-1})/\bar{T}_i\}$ *where $\bar{T}_i$ is some average temperature* appropriate to zone *i*. Whatever its exact definition, $\bar{T}_i$ will be closely proportional to the zonal radiative temperature, T_{ei}; and since the $x_i - x_{i-1}$ are exactly equal to the net radiative energy inputs, $F_{Si} - F_{Li}$, from energy balance considerations, E_1 will be closely proportional to E. In other words a minimum in the external entropy exchange rate, E, corresponds to a minimum in the internal production rate, E_1. Either or both can be minimized to obtain the same predicted distributions.

The significance of E_1 in the present context is the possible analogy of the overall process of meridional energy flow to that of ordinary heat conduction and convection in situations where non-linear terms can be neglected. It is well known that such phenomena are governed by a minimum principle which is easily derived from the basic law of heat transport (e.g. Palm 1972). It states that thermal dissipation always decreases to the minimum which is attained at steady state. Further, it can be shown that entropy production is a minimum for these steady state conditions.

It appears therefore that the present results may have a sound physical basis which can be pursued by more detailed analysis of the internal processes of the system. However, the agreement with observation is far from being a proof of the validity of the overall concept. The most that can be said at this time is that minimization of the rate of entropy production *so defined* appears to be a useful parameterization of the system on the scale to which it has been applied. As the resolution of the model is increased, presumably there will be a point at which the parameterization no longer holds. Hopefully this point will occur at relatively small scales.

It so happens that the absolute magnitude of the minimum obtained for E is quite close to zero (approximately 10^{-2} when the flux units are in mW cm^{-2}), but E is in fact negative. Presumably the sign is irrelevant in view of the accuracy of the parameters used in the overall model, but it is reasonable that this particular entropy production rate should be negative since the system's boundary does not include the energy source.

Despite the complexity of the function determining E, no evidence for the existence of double or multiple minima has been found. The simplex method is capable of finding such minima by alteration of the initial step sizes and simplex size, but in the many trials so far no ' extra ' minima have been found. In other words the model does not appear to predict any form of climate intransitivity.

6. Possible applications

(a) *Alteration of external parameters*

Table 2 gives the computed changes in the zonal average temperature, cloud cover and meridional flux for a 10% increase in the solar constant, R_0, the atmospheric absorptions, m_i, and the atmospheric reflections. This last was achieved by adding 10% of the clear sky albedos to both the g_i themselves and to the cloud albedos, $(d_i - a_i)$.

An increase in solar constant gives a marked increase in the surface temperature of all zones with a global average rise of 0·7K per percent. The change in cloud cover is negligible. Note that this result is in direct contrast to the earlier simplistic calculation by Paltridge (1974) who computed the global average ΔT and $\Delta\theta$ for 1% change in R_0 to be 0·3 K and 0·01 respectively. An increase in atmospheric absorption or reflection decreases the temperature at all latitudes but increases the cloud cover. All three changes slightly increase the equator-pole temperature gradient, but only in the case of increased solar constant are the meridional energy fluxes changed in the obvious direction.

TABLE 2. COMPUTED CHANGES IN ZONAL AVERAGE TEMPERATURE, CLOUD COVER AND MERIDIONAL ENERGY FLUX FROM PRESENT DAY CONDITIONS (FIRST COLUMN) WITH 10% INCREASE IN SOLAR CONSTANT, ATMOSPHERIC ABSORPTION, AND ATMOSPHERIC REFLECTION

Latitude	Normal value	Change with 10% increase of R_0	m_i	d_i and g_i
			Temperature (K)	
64°N	265·8	6·9	−0·60	−2·10
44°	285·7	7·1	−0·57	−1·87
30°	294·7	7·2	−0·50	−1·67
17°	299·1	7·3	−0·50	−1·73
6°	301·1	7·3	−0·43	−1·63
6°	301·2	7·2	−0·47	−1·70
17°	299·6	7·1	−0·53	−1·67
30°	295·5	7·1	−0·57	−1·63
44°	287·3	6·7	−0·67	−1·73
64°S	268·1	5·9	−0·87	−2·03
	Mean:	7·0	−0·57	−1·77
			Cloud cover	
64°N	0·76	0·001	0·020	0·014
44°	0·55	0·000	0·022	0·009
30°	0·45	−0·001	0·022	0·004
17°	0·45	−0·002	0·021	0·003
6°	0·48	−0·001	0·021	0·004
6°	0·48	0·001	0·022	0·005
17°	0·43	0·001	0·021	0·005
30°	0·45	0·001	0·022	0·005
44°	0·55	−0·001	0·022	0·005
64°S	0·76	−0·002	0·020	0·008
	Mean:	0·000	0·021	0·006
		Southward meridional flux (mW cm^{-2})		
53°N	−5·59	−0·68	0·04	0·03
37°	−6·50	−0·81	0·03	−0·02
23°	−4·10	−0·55	0·01	−0·05
11°	−1·47	−0·24	0·00	−0·04
0°	−0·006	−0·06	−0·01	−0·04
11°	1·53	0·05	−0·03	−0·08
23°	4·53	0·30	−0·05	−0·14
37°	6·87	0·51	−0·08	−0·17
53°S	5·94	0·47	−0·08	−0·14

On the basis of this model, it does not appear possible to have a stable ' iced-over ' world as suggested by Budyko (1969). Tests were conducted in which the g_i of all zones were set at the planetary albedo of ice and the oceanic fluxes were set at zero. If any zone exceeded a temperature of 271K (the approximate melting point of ice in sea water) then that zone was restored to its normal albedo and its normal oceanic flux was allowed. The model was re-run until stationary conditions prevailed. It was found that, provided the planetary albedo of ice did not exceed 0·7, the equatorial zones ' melted ' on the first iteration. During subsequent iterations the ice retreated to the point where it could be sustained only in the extreme zones poleward of 53°. The precise theoretical limit of the ice sheets within these extreme zones cannot be established with the model set at its present resolution. Note that even if the surface albedo of ice is as high as 0·8, atmospheric absorption ensures that the planetary albedo is only about 0·6.

A trial of the model where the albedos of the polar zones were assumed to be ' unknowns ' in addition to the meridional fluxes proved unsuccessful. No minimum was found.

(b) Parameterization of the annual cycle

Direct application of the minimum principle assumes steady state conditions. The following is an example to show that a simple parameterization of the ocean time constant may allow a reasonable prediction of the seasonal oscillation in climate.

The movement of the sun north and south of the equator was simulated on a time step Δt of 1 month by appropriate alteration of the ξ_i. At each latitude $\xi_i = \pi/\cos(\phi + D)$, where D is the solar declination for that time of year. Initially the earth was assumed to be in equilibrium with the sun overhead at the equator. The sun was then moved to its new declination. New equilibrium conditions were calculated for each zone using the minimum principle so that a change in temperature ΔT_i of each zone could be computed. The actual temperatures were allowed to alter only by an amount $\Delta T_i\{1 - \exp(-\Delta t/\tau_i)\}$. The τ_i were time constants proportional to the area of ocean in zone i. The sun was moved again and the iteration was allowed to proceed for several 'years' in order to establish stationary conditions. The annual variability so obtained when the '100%-ocean' time constant was set at 150 days is plotted in Fig. 4. Again the agreement with observation is

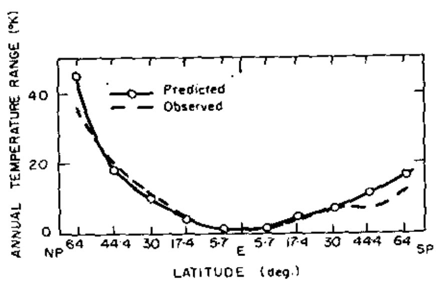

Figure 4. Predicted annual variability of surface temperature as a function of latitude on the basis of a zonal time constant proportional to the zonal fraction of ocean-area. The '100%-ocean' time constant is 150 days.

good – especially since no attempt was made in this simple example to change the zonal constants to values appropriate for the different solar declinations. Because of this neglect, the annual mean temperatures were slightly lower ($\approx$ 1–2 K) than the equilibrium values of Fig. 3(a).

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